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Estimating Mercury's 88-day libration amplitude from orbit

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Abstract

In this paper it is derived that the libration of Mercury can be described by $\gamma = \Phi_0 \sin M + \Phi_0/K \sin 2M$ where Φ_0 is the unknown libration amplitude, M is Mercury's mean anomaly and $K = -9.483$. Φ_0 can be determined by comparing pairs of images of the same landmarks taken by an orbiter at different positions of Mercury. If the angle between the orbit plane of a polar orbiter and Mercury's line of periapsis is between -60° and 60° and if one landmark at the equator is imaged per day with a relative precision of 1.6 arcsec, then the libration amplitude can be determined in two Mercury years (176 days) with an accuracy of 1 arcsec or better, which is sufficient to answer the question whether Mercury has a solid or fluid core.

Keywords: Mercury; BepiColombo; Libration amplitude; Spin-orbit resonance

1. Introduction

The discovery of an intrinsic magnetic field for Mercury by the Mariner 10 flybys in 1974 and 1975 (Ness et al., 1974) can be best explained by a hydromagnetic dynamo in a fluid outer core. Peale (1988) has proposed that the radius of the expected fluid core can be determined by measuring the amplitude of the 88-day forced libration in longitude, the obliquity (inclination of the equator to Mercury's orbital plane) and second-degree gravity potential coefficients. Let A, B and C be the principal moments of inertia of Mercury, C_m the mantle's moment of inertia, M_M the mass of Mercury, a_e its equatorial radius, then

$$\frac{C_m}{C} = \left(\frac{C_m}{B-A}\right)\left(\frac{B-A}{M_M a_e^2}\right)\left(\frac{M_M a_e^2}{C}\right). \quad (1)$$

A value of C_m/C of 1 would indicate a core firmly coupled to the mantle and most likely solid. If the entire core or the outer part is fluid, $C_m/C \simeq 0.5$ for the large core size ($r_c \simeq 0.75$) as modelled by Cassen et al. (1976).

The third factor in Eq. (1) can be derived from the obliquity, C_{20} and C_{22} (see Peale, 1988), the second factor is equal to $4C_{22}$ and the first factor is approximately the inverse of the libration amplitude

$$\phi_0 = \frac{B-A}{C_m}. \quad (2)$$

We assume that the gravitational harmonics and the obliquity will be determined by the BepiColombo mission (Novara, 2002) to a few percent or better accuracy, so the uncertainty in the determined value of C_m/C will be dominated by the uncertainty in the libration amplitude. BepiColombo's objective is to determine both, obliquity and libration amplitude, but in this paper it is only assessed how accurate the libration amplitude can be determined with a camera on board a spacecraft in a 400×1500 km polar orbit.

2. Mercury's 88-day libration

Before observing the libration of Mercury, the principal dynamics have to be understood. The differential equation which governs the libration is taken from Balogh and Giampieri (2002). The Sun exerts a torque along Mercury's spin axis, with magnitude

$$T = -\frac{3GM_\odot}{2r_p^3}(B-A)\sin 2\psi, \quad (3)$$

where $M_\odot$ is the mass of the Sun, r_p the Sun–Mercury distance, and ψ the angle between Mercury's equatorial long

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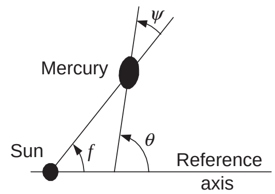

Fig. 1. The geometry of the torque. Mercury's long axis makes an angle θ with the inertial reference axis, and an angle ψ with the direction to the Sun. $f = \theta - \psi$ is the true anomaly of Mercury (Balogh and Giampieri, 2002).

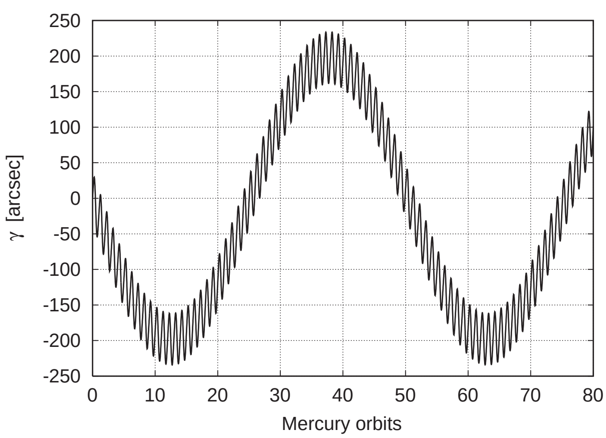

Fig. 2. Libration angle γ as function of time calculated by numerical integration of Eq. (6). For $\alpha \equiv \frac{3}{2}(B-A)/C_m$ we take 3×10^{-4} (consistent with Peale's papers) and the initial conditions are chosen arbitrarily: $\gamma_0 = 0$ and $\dot{\gamma}_0 = 1.6$ arcsec/day.

axis and the direction of the Sun (see Fig. 1). From Fig. 1, the rotation angle in inertial space is $\theta = f + \psi$, where f is Mercury's true anomaly.

Assuming that the core does not follow the libration of the mantle, this torque T induced by the Sun gives the rate of change of the mantle's spin angular momentum L

$$\frac{dL}{dt} = C_m \ddot{\theta} = T. \tag{4}$$

Since Mercury's spin is in a 3:2 resonance with the orbital frequency, we introduce the libration angle as

$$\gamma \equiv \theta - \frac{3}{2}M = \psi + f - \frac{3}{2}M, \tag{5}$$

where M is Mercury's mean anomaly. From Eq. (3) to (5) it is a cinch to obtain

$$\ddot{\gamma} = -\frac{3GM_\odot}{2r_p^3}\left(\frac{B-A}{C_m}\right)\sin(2\gamma + 3M - 2f). \tag{6}$$

The derivation of Eq. (6) can also be found in Murray and Dermott (1999) and Celletti (1990). A more general set of differential equations including the coupling with the (different) libration of the core is given by Peale et al. (2002).

We neglect the coupling with the core libration and numerically integrate Eq. (6). The results are seen in Fig. 2.

Table 1
Analytical expansion and numerically computed values ($e = 0.2056$) for the eccentricity functions G_{20q}. For G_{203} no analytical expansion was found in the literature.

q	G_{20q}	Value for $e = 0.2056$
−1	$-e/2 + e^3/16 + \cdots$	−0.1023
0	$1 - 5e^2/2 + 13e^4/16 + \cdots$	0.8957
1	$7e/2 - 123e^3/16 + \cdots$	0.6542
2	$17e^2/2 - 115e^4/6 + \cdots$	0.3261
3		0.1380

There are two major components of Mercury's libration: a long-term libration with a period of some years (depending on $\alpha \equiv \frac{3}{2}(B-A)/C_m$) and a short-term libration with a period of 88 days, i.e. one Mercury year.

Balogh and Giampieri replaced in Eq. (6) the following relation between the distance from Mercury to the Sun r_p, the semimajor axis a and the so-called eccentricity functions G_{20q} (see Kaula, 1966)

$$\left(\frac{a}{r_p}\right)^3 \sin(2\gamma + 3M - 2f) = \sum_q G_{20q}(e)\sin[2\gamma + (1-q)M] \tag{7}$$

and obtained (after substituting the derivatives with respect to time by derivatives with respect to M):

$$\gamma'' + \alpha \sum_q G_{20q}(e)\sin[2\gamma + (1-q)M] = 0. \tag{8}$$

Averaging Eq. (8) over one orbital period, Balogh and Giampieri determined the period of the long-term libration to be about $(2\alpha G_{201})^{-1/2}$ revolutions. For our α of 3×10^{-4} and G_{201} as given in Table 1 this long-term period is 50.5 revolutions, i.e. 12 years. Due to the finite tidal dissipation function Q this libration should have damped out and no observable amplitude should have remained, but "we have been surprised in the past" as Peale et al. (2002) warns. Nevertheless we assume in this paper that there is no long-term libration left.

What remains to be determined is the amplitude Φ_0 of the 88-day libration, which gives the clue to the internal structure of Mercury. Integrating Eq. (8) for small values of γ we get

$$\Phi_0 = \alpha(G_{200} - G_{202}). \tag{9}$$

If $\alpha = 3 \times 10^{-4}$ (a value close to the one used by Peale (1988)), then $\Phi_0 \simeq 35$ arcsec. There are also higher-frequency terms which need to be included to describe Mercury's libration more precisely, as can be seen in Fig. 3. We confine ourselves to the following approximation:

$$\gamma = \Phi_0 \sin M + \Phi_1 \sin 2M. \tag{10}$$

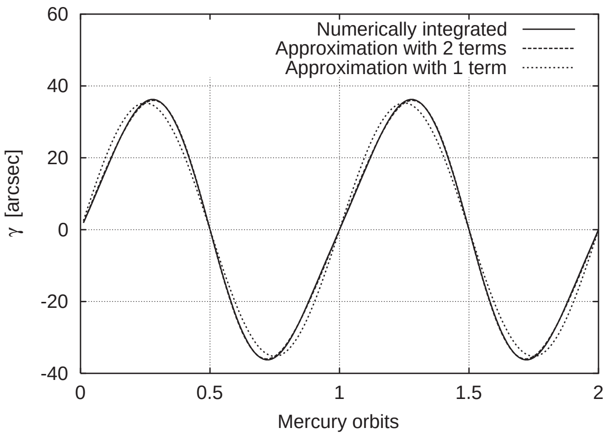

Fig. 3. Mercury's libration γ can accurately be described by two sine terms: $\Phi_0 \sin M + \Phi_1 \sin 2M$ (the solid line and dashed line can hardly be discerned). If only a single sine function is used as approximation the best fit (i.e. minimum least squares) has a 2% smaller amplitude and the maximum/minimum libration angles appear 11° before/after the correct maximum/minimum which are at 101°/259° mean anomaly (123°/237° true anomaly). Note that these values are independent of the initial conditions for γ and $\dot{\gamma}$.

Φ_1 can be obtained if we include $G_{20q}(e)$ up to order $q=3$ in Eq. (8) and integrate

$$\Phi_1 = \frac{\alpha}{4}(G_{201} - G_{203}) \tag{11}$$

which gives $\Phi_1 \simeq -3.7$ arcsec. Note that the relation between the two amplitudes $K = \Phi_0/\Phi_1 = -9.483$ does not depend on α. The approximation with two terms nicely matches the numerical solution (see Fig. 3).

3. Observation geometry from orbit

To get a feeling for the observability of the libration of Mercury, this relation helps: a libration amplitude of 10 arcsec causes a landmark at the equator of Mercury ($r = 2440$ km) to drift regularly to the East and to the West up to 118 m with respect to its 'nominal' position (where nominal refers to a Mercury with a constant spin rate). If $\alpha = 3 \times 10^{-4}$, Φ_0 is 35 arcsec and all landmarks will show a libration of plus/minus 400 m at the equator.

The basic idea how to observe the libration is to take images of the same landmarks at different true anomalies of Mercury. Two pictures of the same landmark are overlaid and, with pattern matching techniques, the displacement of the surface of one image with respect to the other image is determined. Jorda and Thomas (2000) simulated the pattern matching of albedo features and craters for a Mercury orbiter and concluded that sub-pixel accuracy can be achieved, if images obtained at phase angles $< 5°$ and $> 55°$ are excluded (for albedo spots) and if the difference between the phase angles are smaller than 35° (for craters).

An important factor which determines the observability of Mercury's surface is the right ascension of ascending node of the target orbit. BepiColombo is in a polar orbit and thus, due to Mercury's very small obliquity, its orbital plane is perpendicular to Mercury's orbital plane. Therefore, it is convenient to define the angle β as the angle between the target orbital plane and Mercury's line of periapsis. β is illustrated in Fig. 4.

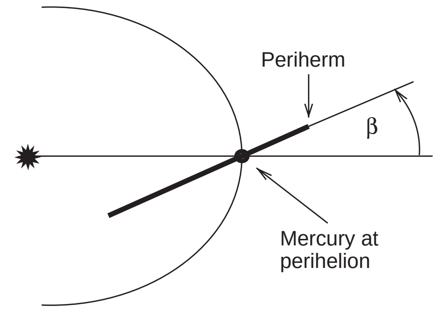

Fig. 4. Definition of the angle β between the target orbital plane (which is polar and plotted here as a thick line) and Mercury's line of periapsis.

BepiColombo's orbit has a periherm altitude of 400 km and an apoherm altitude of 1500 km. The argument of periherm is about 180° (or 0°), i.e. periherm and apoherm are close to the equator. Since the libration can be best observed at the equator, landmark images should be taken there. The resolution of the images is inversely proportional to the altitude and thus it is at least three times lower when taken at apoherm. Thus, it is evident that images taken around periherm will provide a better accuracy of the displacement of surface features. If β is 0°, then low-altitude images can be taken during a long time around aphelion. On the contrary if β is 180°, the part of Mercury's surface which can be imaged from low altitude is much smaller, because Mercury spends less time around perihelion. Fig. 5 shows the surface coverage during two aphelion passages of Mercury for $\beta = 0°$.

Fig. 5 reveals that $\beta = 0°$ provides benign conditions to observe the same longitude intervals at two different true anomalies of Mercury. If we pick the longitude of 0°, we have the first images taken on day 75 (true anomaly = 222°, mean anomaly = 240°) with a phase angle of 41° (sun elevation of 49° corresponding to "09:16 local time"). A second image is taken on day 133 (true anomaly = 138°, mean anomaly = 120°) with a phase angle of 43° (corresponding to "14:52 local time").

In our subsequent analysis we assume that landmarks are omnipresent, images are taken only at the equator when the spacecraft altitude is below 1000 km and images taken with a phase angle larger than 60° are rejected. With these assumptions the longitude intervals from 152° to 211° and from −30° to 28° can be covered twice if $\beta = 0°$. Fig. 6 shows the corresponding mean anomalies where these longitude strips are imaged. The first images are taken at a mean anomaly around 240°, shortly before the libration is at its minimum and the second images of the same longitude band is taken at a mean anomaly around 120°, shortly after the

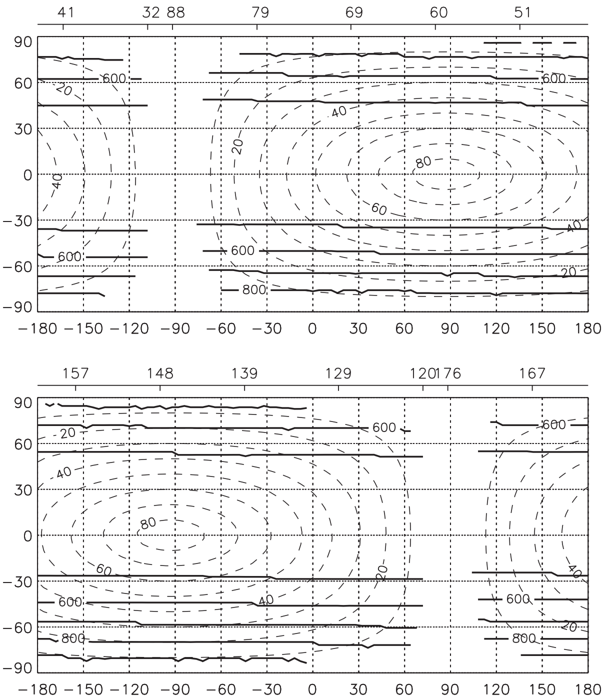

Fig. 5. BepiColombo's coverage of Mercury's surface during two consecutive aphelion passages (from Mercury true anomaly 90° to −90°) for $\beta = 0°$. The solid lines show the altitude of the observations and the dashed lines the sun elevation at the sub-satellite point (i.e. 90° minus phase angle). The surface is scanned from East to West. The secondary x-axis on top of the plots represents the time of the observations measured in days (day 0 corresponds to the time when the spacecraft flies over the terminator, at a Mercury true anomaly of −90°. The plot on top covers the period from day 32 to day 88, the plot below from day 120 to day 176).

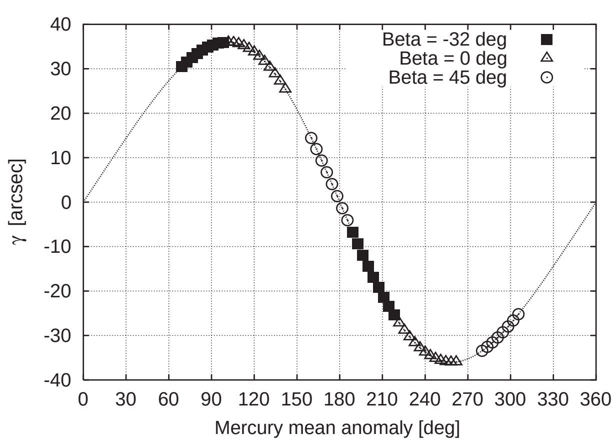

Fig. 6. Positions (mean anomalies) along Mercury's orbit where BepiColombo can observe equatorial surface features from low altitude twice during two consecutive Mercury orbits superimposed on the libration angle (which shall be measured) as function of the orientation of the spacecraft's orbital plane (β).

libration is at its maximum. This is a nearly optimum situation. Also for $\beta = -32°$ (the value which is obtained if the interplanetary trajectory is fuel-optimized, see Katzkowski et al. (2002)) and for $\beta = 45°$ the positions in the orbit of Mercury where the images can be taken are located such that the libration can be observed. For $\beta = 90°$, however, there is not a single opportunity to image the same longitude twice if the constraints on the phase angles have to be fulfilled.

4. Achievable accuracy as function of β

In this section a straightforward covariance analysis of the error in the libration amplitude will be presented. We have a linear system of observations

$$Z = A\ \phi_0 + v, \tag{12}$$

where Z is the vector with the observed displacements of surface features, ϕ_0 is the amplitude to be estimated and v is the error in the observations. The components a_i of the vector A are

$$a_i = \sin M_{1i} - \sin M_{2i} + \frac{1}{K}(\sin 2M_{1i} - \sin 2M_{2i}), \tag{13}$$

where M_{1i} is Mercury's mean anomaly where the landmark i is imaged the first time and M_{2i} is Mercury's mean anomaly where it is imaged the second time.

The least-squares estimate of the libration amplitude is

$$\hat{\phi_0} = \phi_0 + \frac{A^{\mathrm{T}} v}{A^{\mathrm{T}} A}. \tag{14}$$

We assume that the components v_i of the measurement error vector are independent from each other and Gaussian white noise with the same standard deviation σ because it turned out that all observations are taken at very similar altitudes below 500 km. At this altitude 1 pixel uncertainty in the pattern matching corresponds to 12.5 m on the surface. On top of this uncertainty there are three other error sources (Milani et al., 2001):

- The error in the knowledge of the position of the spacecraft in a mercurycentric nonrotating frame is estimated to be < 10 m (RMS value).
- The error in the knowledge of the pointing of the star mapper with respect to an absolute reference frame is better than 2 arcsec corresponding to 5 m on the surface from an altitude of 500 km.
- The error in the angles defining the pointing of the camera with respect to the pointing of the star mapper is 3 arcsec corresponding to 7.5 m on the surface.

Since the four errors are all independent from each other, the total RMS error is

$$\sigma = \sqrt{12.5^2 + 10^2 + 5^2 + 7.5^2} = 18.4 \text{ m}, \tag{15}$$

which corresponds to an error in longitude of 1.6 arcsec at the equator of Mercury.

Using the relation

$$\mathrm{Var}(a_1 v_1 + a_2 v_2) = (a_1^2 + a_2^2)\sigma^2,$$

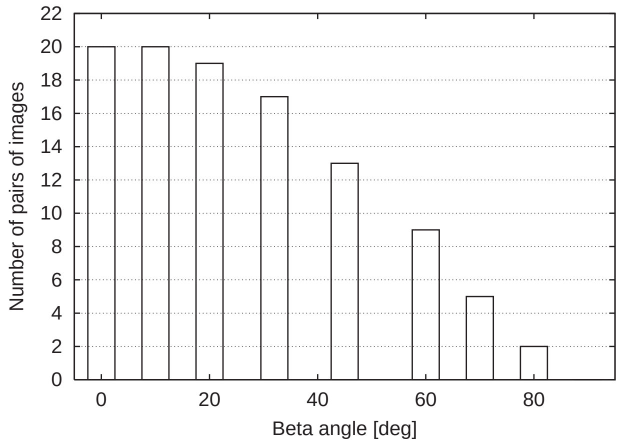

Fig. 7. Total number of pairs of landmark images which can be obtained in 2 Mercury years fulfilling the constraints on the altitude (below 1000 km) and the phase angle ($< 60°$) as function of the β-angle. Note, the two images which are compared are always taken at a very similar altitude and the differences in their phase angles are always smaller than 35° (see Fig. 5).

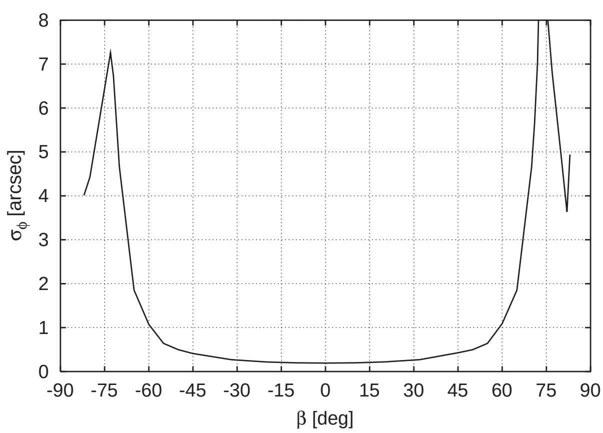

Fig. 8. Standard deviation of the estimate of the libration amplitude $\hat{\phi_0}$ as a function of β.

the standard deviation of the estimate of the libration amplitude $\hat{\phi_0}$ can be derived from Eq. (14). The result is

$$\sigma_\phi = (A^T A)^{-1/2}\sigma. \qquad (16)$$

The same result was also obtained by Bierman (1977, p. 16) in a more general case.

Now we assume that we observe one landmark per day and determine $A^T A$ as a function of β for a total mission duration of 176 days (2 Mercury years). Fig. 7 shows the number of pairs of images acquired during this period for different values of β. As mentioned before, for $\beta = 90°$ there are no pairs of images satisfying the constraints.

Fig. 8 shows the uncertainty in the estimation of the libration amplitude as a function of β. In general, the uncertainty increases as the number of images decreases. However, also Mercury's mean anomaly where the images are taken plays a role. For instance, for $\beta \simeq 73°$ the libration angles in the

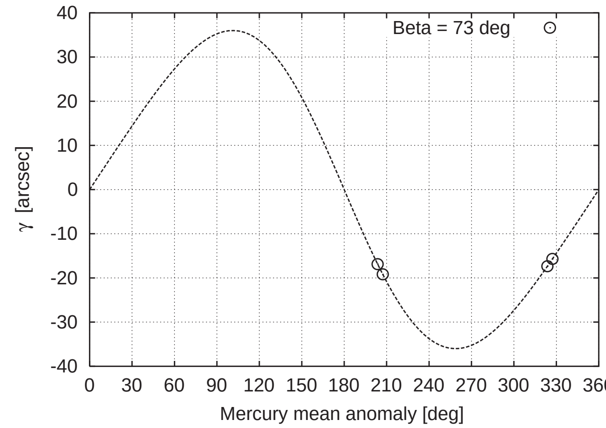

Fig. 9. Positions (mean anomalies) along Mercury's orbit where the same landmarks can be imaged twice for $\beta = 73°$. The libration angle γ is about the same for two corresponding images and thus it is difficult to determine the libration amplitude.

pairs of images are nearly identical and do not allow the estimation of the libration amplitude, as can be seen in Fig. 9.

5. Discussion

For β around 0° the calculations above give an error in the amplitude determination which is below 1%. This is much better than what Wu et al. (1997) and also Smith et al. (2001) found. The analysis by Wu et al. (1997) like the one presented here is based on comparing images taken from the BepiColombo orbit, but they assume larger measurement errors in their simulations (e.g. 15 arcsec for the attitude uncertainty of the optical axis) and assume images taken at 30° latitude (rather than at the equator). This results in a total uncertainty of 3.8 arcsec in the libration angle compared to the 1.6 arcsec which we assume. Wu et al. (1997) did not perform a full simulation including the number of possible observations and thus, the 3.8 arcsec are considered also as the overall achievable accuracy in the libration amplitude. In our approach the uncertainty tends to zero when the number of observations tend to infinity because we have no biases and system noise in our equations.

Smith et al. (2001) simulate the estimation of the libration amplitude by remote sensing of the gravity field and altimetry from the MESSENGER orbit (200 × 15 200 km). Based on their assumptions the overall achievable accuracy in the libration amplitude is 25 m at the equator corresponding to 2.1 arcsec. However, the MESSENGER orbit is inferior to the BepiColombo orbit if the objective is to determine the gravity field and the rotation state of Mercury.

A complete numerical analysis is necessary to assess the accuracy in the libration amplitude which can be achieved with the BepiColombo mission. The analysis presented here neglects systematic noise and therefore gives too optimistic results although the total mission duration will be 4 Mercury years instead of 2 years and probably more than one useful image per day will be collected. The covariance analysis performed by Sánchez Ortiz et al. (2003) includes systematic noise. It shows that 0.3 arcsec accuracy can only be achieved if the camera has a nadir offset pointing capability of 1° and if the number of observable landmarks is large (about 700 over the surface of Mercury). When there are 10 times fewer landmarks, the achievable accuracy will be worse than 1 arcsec.

6. Conclusions

On top of Mercury's continuous rotation which is in a 3:2 resonance with Mercury's orbital period, there is a libration motion which can be roughly described by a single sine function. However, for more precise analysis like the one performed in this paper, higher order terms must be included. It was shown that the libration can be well approximated by $\Phi_0 \sin M + \Phi_0/K \sin 2M$ where Φ_0 is the unknown libration amplitude and $K = -9.483$ which is independent of Φ_0.

Φ_0, the amplitude of Mercury's libration, can be determined with high accuracy if the angle between the orbit plane of a polar orbiter and Mercury's line of periapsis (β-angle) is between $-60°$ and $60°$. For $\beta = 0°$ an accuracy of below 0.3 arcsec can be achieved if pattern matching techniques are applied to 20 independent pairs of surface images which will be available in 2 Mercury years (176 days).

However, no systematic noise was considered. A full numerical analysis was carried out by Sánchez Ortiz et al. (2003) simulating at the same time the orbit determination process and the determination of the libration amplitude. Their results confirm that—if the β-angle is chosen properly and if the camera has a 1° pointing capability—the libration amplitude can be determined with an accuracy which is sufficient to answer the question whether Mercury has a fluid or solid core.

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References

Balogh, A., Giampieri, G., 2002. Mercury: the planet and its orbit. Rep. Prog. Phys. 65, 529–560.

Bierman, G.J., 1977. Factorization Methods for Discrete Sequential Estimation. Academic Press, New York.

Cassen, P.M., Young, R.E., Schubert, G., Reynolds, R.T., 1976. Implications of an internal dynamo for the thermal history of Mercury. Icarus 28, 501–508.

Celletti, A., 1990. Analysis of resonances in the spin–orbit problem in celestial mechanics: the synchronous resonance. J. Appl. Math. Phys. 41, 174–204.

Jorda, L., Thomas, N., 2000. The accuracy of pattern matching techniques for the radio science experiment of ESA's Mercury Cornerstone mission. Preliminary study at Max-Planck Institute for Aeronomie, Katlenburg-Lindau, Germany (not published).

Katzkowski, M., Corral, C., Jehn, R., Pellon, J.-L., Landgraf, M., Khan, M., Yáñez, A., Biesbrock, R., 2002. BepiColombo Mercury Cornerstone Mission Analysis, MAS working paper No. 432, ESA/ESOC, Darmstadt, Germany.

Kaula, W.M., 1966. Theory of Satellite Geodesy. Blaisdell, New York.

Milani, A., Rossi, A., Vokrouhlický, D., Villani, D., Bonanno, C., 2001. Gravity field and rotation state of Mercury from the BepiColombo radio science experiments. Planet. Space Sci. 49, 1579–1596.

Murray, C.D., Dermott, S.F., 1999. Solar System Dynamics. Cambridge University Press, Cambridge.

Novara, M., 2002. The BepiColombo ESA cornerstone mission to Mercury. Acta Astronautica 51, 387–395.

Ness, N.F., Behannon, K.W., Lepping, R.P., Whang, Y.C., Schatten, K.H., 1974. Magnetic field observations near Mercury. Science 185, 151–160.

Peale, S.J., 1988. Rotational dynamics of Mercury and the state of its core. In: Vilas, F., Chapman, C., Matthews, M. (Eds.), Mercury. University of Arizona Press, Tucson, pp. 461–493.

Peale, S.J., Phillips, R.J., Solomon, S.C., Smith, D.E., Zuber, M.T., 2002. A procedure for determining the nature of Mercury's core. Meteorit. Planet. Sci. 37, 1269–1283.

Sánchez Ortiz, N., Belló Mora, M., Jehn, R., 2003. BepiColombo mission: estimation of Mercury gravity field and rotation parameters, 54th International Astronautical Congress, Bremen, Germany, paper IAC-03-Q.2.07.

Smith, D.E., Zuber, M.T., Peale, S.J., Phillips, R.J., 2001. Estimating the libration of mercury by remote sensing of gravity and altimetry. In: Mercury: Space Environment, Surface, and Interior, Lunar and Planetary Institute Contribution No. 1097.

Wu, X., Bender, P.L., Peale, S.J., Rosborough, G.W., Vincent, M.A., 1997. Determination of Mercury's 88 day libration and fluid core size from orbit. Planet. Space Sci. 45, 15–19.