OPTIMIZATION OF LOW-ENERGY RESONANT HOPPING TRANSFERS BETWEEN PLANETARY MOONS

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ABSTRACT
In response to the scientific interest in Jupiter’s moons, NASA and ESA have recently decided to target in priority Jupiter’s largest moons for the next outer planet flagship missions. For those missions inter-moon transfers of the Jovian system offer obvious advantages in terms of scientific return, but are also rather challenging to design and optimize due in part to the hostile environment of third body perturbations. The approach outlined in this paper confronts this shortcoming by exploiting the multi-body dynamics with a patched three-body model to enable multiple “resonant-hopping” gravity assists. Initial conditions of unstable resonant orbits are pre-computed and provide starting points for the elusive initial guess associated with the highly nonlinear optimization problem. The core of the optimization algorithm relies on a fast and robust multiple-shooting technique to provide better controllability and reduce the sensitivities associated with the close approach trajectories. The complexity of the optimization problem is also reduced with the help of the Tisserand-Poincaré (T-P) graph that provides a simple way to target trajectories in the patched three-body problem. Preliminary numerical results of inter-moon transfers in the Jovian system are presented. For example, using only 59 m/s and 158 days, a spacecraft can transfer between a close resonant orbit of Ganymede and a close resonant orbit of Europa.

I. INTRODUCTION
The exploration of the planetary moon system of Jupiter was set jointly by NASA and ESA as the priority for the next flagship class tour and orbiting mission, officialized recently as the Europa Jupiter System Mission (ESJM). In fact, referred to as a miniature solar system, the Jovian system has recently been attracting much scientific attention, with a particular emphasis on the four Galilean moons: Io, Europa, Ganymede, and Callisto. A vast water ocean may exist beneath Europa’s surface, and heat provided by tidal flexing (aroused from the orbital eccentricity and resonance between moon orbits) ensures that the ocean remains liquid. Ganymede and Callisto are now also thought to have vast hidden oceans beneath their crusts. This presence of liquid water naturally raises the question of the habitability of life on Jupiter’s moons. In addition, the dynamical mechanism of the Jupiter system and its conditions of formation remain mysterious. To address all these key unknowns, the baseline EJSM consists of two platforms operating in the Jovian system: the NASA-led Jupiter Europa Orbiter (JEO), and the ESA-led Jupiter Ganymede Orbiter (JGO). JEO will perform several fly-bys of Io before settling into orbit around Europa. Following a similar approach, JGO will perform an in-depth exploration of the Ganymede-Callisto pair and then orbit Ganymede. This multiplatform approach can therefore provide the basis for an in-depth comparative study of Ganymede and Europa.

In the recent years both NASA and ESA conducted a variety of mission options to achieve a planetary moon tour at Jupiter. A very challenging part of the trajectory design of this type of mission is the
orbital transfer from one planetary moon to another, which is an important phase for the JEO and JGO orbiters. The complexity of the trade space, the heavy dependence on the three-body regimes of motion, and the very limited fuel budget contribute to the challenging design problem. The difficulty is especially true when the spacecraft is in a regime of high Jacobi constant or low three-body energy, which is preferred for cheap escape and capture maneuvers. This low energy precludes the use of the well-known Tisserand graph to design a patched conic, Galileo-style tour of the satellites because such transfers would not be in the feasible domain of the graph. In the most recent traditional approaches, transfers are computed using Vinfinity Leveraging Maneuvers (VILM). In the well-studied VILM problem, a relatively small deep-space maneuver in conjunction with a gravity assist at the body is used to efficiently modify the spacecraft relative velocity to the body. While this strategy has resulted in many successful missions, the solution space is limited since it relies on the dynamical basis of the two-body problem with a zero radius sphere of influence.

To design a more efficient inter-moon transfer, a multi-body approach can be taken instead. Recent applications of dynamical systems theory to the multi-body astrodynamics problem have led to a new paradigm of trajectory design. From this perspective, trajectories can take advantage of natural dynamics to efficiently navigate in space rather than ‘fighting’ the dynamics with thrusting. A number of recent ‘multi-moon orbiter’ papers demonstrate the impressive ΔV savings that can be obtained for moon transfers when exploiting the multi-body dynamics. One possibility extensively studied by many authors is the use of invariant manifolds of libration point orbits and unstable periodic orbits of the three-body problem. Those manifolds form a transportation tube network that naturally provides transit trajectories between the bodies. However, this approach is not intuitive and requires extensive computations to generate all the manifolds and find their connections.

Another multi-body approach that recently emerged is the employment of multiple three-body resonant gravity assists. Undoubtedly related to the invariant manifolds, the three body gravity assists are the key physical mechanisms that allow the spacecraft to jump between orbital resonances (‘resonant hopping’). By analogy with the concept of the Interplanetary Superhighway popularized by Lo, the resonance hopping mechanism can be seen as part of an Intermoon Superhighway. This phenomenon can steer the orbital energy to achieve desired transfers with a significant reduction in propellant requirements. However, existing multi-body approaches generally do not include a rigorous and systematic optimization procedure. Fuel-efficient trajectories have been previously obtained through tedious trial-and-errors, or computational intensive global searches on discretized control inputs with analytic approximations to the dynamics.

In this paper, we intend to optimize low-energy (i.e. quasi-ballistic) resonant hopping transfers in the patched three body problem between arbitrary resonances of two moons. To meet that goal, the focus of our work will be twofold: 1) understand the resonant hopping mechanism and construct families of periodic resonant orbits for arbitrary three-body systems as initial guesses for the hypersensitive optimization problem; 2) develop a new systematic methodology to select promising resonant paths and optimize the resulting transfers using a fast and robust multiple shooting method. We leverage this work on recent advancements in the mission design applications of dynamical systems theory. The ultimate goal is to develop systematic practical methods for finding repeatable, three-dimensional, ephemeris model trajectories. While the main applications of this study consider transfers in Jovian system, the framework is established in a general manner in order to apply to a variety of proposed planetary moon missions.

For this paper, we consider a transfer between two moons only (referred to as the outer moon and the inner moon) of an arbitrary celestial system with a primary and two bodies (moons) in circular co-planar orbits. We intend to optimize a trajectory from the outer moon to the inner moon (this order is arbitrary). A patched three-body approach is adopted, where a dynamical model representing a planetary system is decomposed into a series of perturbed three-body problems. In each three-body sub-problem, the key idea is to target a predetermined sequence of resonant periodic orbits through small impulsive maneuvers. During the optimization process, directing a trajectory towards a target is difficult due to the high numerical sensitivity that results from the unstable and chaotic dynamics. A multiple shooting technique is therefore employed that splits the integration interval to limit error propagation and reduce sensitivity accumulation. The resulting optimization problem is then treated using a state-of-the-art NLP solver.

Targeting successive resonant orbits is also facilitated via a new tool, the Tisserand-Poincaré (T-P) graph, an extension of the Tisserand graph to the three-body problem. Using the T-P graph, the targeting problem for ballistically connecting two orbits between patched
three-body models can be reduced to two uncoupled single dimension problems (e.g. one intersection point in the T-P graph).

To navigate the immense chaotic design space, a good initial guess of the optimal trajectory that takes advantage of the natural dynamics of the problem is generated. We use the Keplerian Map, a simplified analytical approach in the restricted three-body problem that approximates the impulse provided by the perturbing moon during a flyby. This allows for quick, analytic explorations of the design space in order to identify promising feasible and efficient resonant sequence paths.

The paper is organized as follows. First, we present briefly the three-body resonant gravity-assist mechanism. Then, we initiate an initial guess strategy to find promising candidate resonant orbits to target. In the next part, a general resonant orbit generator is designed and implemented to directly exploit the chosen resonant path. The following section is the bulk of the paper where we explain in details the new multiple shooting strategy for moon transfers. Finally, an example application is presented of a transfer between Ganymede and Europa.

### II. MECHANISM OF THREE-BODY RESONANT GRAVITY-ASSIST TRANSFERS

A three-body resonant gravity-assist is a special class of gravity assists (inexplicable with patched conics) which occurs outside of the perturbing body’s sphere of influence and allows the spacecraft to jump between orbital resonances with the planetary moon. When the spacecraft orbit is in resonance with the moon’s orbital period, it can regularly re-encounter the moon, which makes multiple gravity assists possible. Those repeated high altitude flybys provide successive effective velocity impulses (in the form of energy kicks) to perform the transfer and reduce the amount of propellant needed. Throughout the paper, we will characterize a resonant orbit with two numbers $K:L$ or $L:K$ where $K<L$ (see Figure 1). The number on the left represents the number of body revolutions, while the number on the right represents the number of spacecraft revolutions.

Combining orbital resonances with gravity-assists in space mission design dates back to the late sixties, when the Italian researcher Giuseppe Colombo discovered the spin-orbital resonance of Mercury and pointed out to NASA the possibility of a resonant, multiple flyby orbit for the post-encounter trajectory of the Mariner mission. This allowed for a dramatic increase of the science return of the mission. This technique was then considered for repeated flybys of the Earth for modifying a spacecraft trajectory or for a cheap transfer to some near-Earth asteroids. However, all the resonant flybys are performed at relatively low-altitudes and assume pure two-body motion. The three-body, high-altitude resonant gravity-assists are different and have never been implemented as the main dynamics driver in a real mission, with the exception of the recent Smart1 mission to the moon.

![Figure 1 - Phases of Inter-Moon Resonant Gravity Assists](image-url)
The full inter-moon transfer mechanism resulting from three-body resonant gravity-assists is explained schematically in Figure 1. To understand how the transfer is achieved, it is necessary to split up the trajectory into two phases in which only the perturbations due to the dominant moon are considered. In the first portion, the spacecraft decreases its periapsis and jumps between orbital resonances with the outer moon by performing repeated gravity assists when it passes through apoapsis. As we will explain next, a very precise spacecraft/moon geometry is required to achieve repeated resonance hopping. Therefore, small DSMs are added at the corresponding apse before each encounter to provide control over the geometry and avoid the spacecraft getting stuck in prohibitively long resonances. Once the spacecraft periapsis is close to the inner moon radius, the perturbation model switches from the outer moon to the inner moon. The spacecraft orbit where the model transfers is deemed the ‘switching orbit’. The second phase then takes place and the same principle is applied (in reverse) with the inner moon as the perturber.

The mechanism of resonant gravity assists is explained in detail by Ross and Scheeres in Ref. 28. The effect of the moon is to provide a kick to the spacecraft so that it can achieve another orbital resonance. An analytical expression in the form of an energy kick function is derived in the same paper to approximate the impulse provided by the perturbing moon at periapsis. We extended this expression in the case of a flyby at apoapsis (see Appendix 1). Figure 2 gives an example of change in semi-major axis as a function of \( w \), the argument of periapsis in the rotating frame (i.e. the angle between the Jupiter – Moon axis and the Jupiter – spacecraft axis). Notice that the kick function has a large magnitude over very small values of the periapsis angle, so this technique is likely to be very sensitive on the argument of periapsis (which determines the geometry of the flyby). Figure 2 also shows there is little difference between the real kick function computed by integration and the analytical one, which is indicative of the accuracy of the method (at least in the case of a single iteration). Note that it was necessary to do some ‘calibration’ of the initial conditions of numerical propagation to obtain such a good match. The kick function implies that the change in semi-major axis is instantaneous, which is not the case in reality. For a given pair \((w,a)\), it is possible to find the state at the corresponding apse (see Appendix 2 for the detailed numerical procedure). From this state we integrate backward in time (one-third of the orbital period) to find the approximate unperturbed semi-major axis before the flyby. Then in the same way, we integrate forward in time to find the new semi-major axis after the flyby.

As emphasized in Figure 3, the shape of the kick function (odd with respect to the periapsis angle) implies that when the spacecraft passes a little behind (respectively in front of) the moon, then the semi-major axis is instantaneously decreased (respectively increased). The maximum negative kick is at a certain value \( w_{\text{max}} \) (which depends on the parameters of the problem), while the maximum positive kick is at \(-w_{\text{max}}\). It follows that repeatedly targeting the right region (i.e. close to \( w_{\text{max}} \)) using small impulsive maneuvers can produce large changes in semi-major axis and successive resonance hopping.
III. RESONANT PATH SELECTION

Between two given planetary moons, an infinite number of resonant hopping trajectories exists, since a transfer is built by combining multiple resonant orbits (see Figure 1 and Figure 4). The first step of an inter-moon resonant hopping design must therefore consist of finding a good resonant path.

![Figure 4 – Resonant Path](image)

Clearly different resonant paths can lead to large variations in the fuel required to accomplish the transfer, and further each path consists of many local optima. The flight time must be also be considered as many resonant orbits yield very long transfers that are not feasible for a real mission (characterized by strict time constraints).

In the context of the VILM strategy, Brinckerhoff and Russell chose to enumerate all the possible resonant combinations and solve all of resulting problems. But this approach is efficient only because the VILM algorithms are computationally inexpensive. On the contrary, in the case of the three-body approach, the dynamics require expensive numerical integration to propagate the states of the spacecraft.

In this section we describe the quick analytical method introduced by Ross and Scheeres to generate promising resonant paths for the inter-moon orbiter trajectory. The multiple gravity-assist strategy is approximately modeled by the so-called ‘Keplerian Map’ (or periapsis Poincaré Map), which is a recursive relationship of the form:

\[
\begin{align*}
W_{n+1} &= w_n - 2\pi (-2K_{n+1})^{3/2} / \left( K_n + \mu f(w_n) \right) \\
K_{n+1} &= K_n
\end{align*}
\]

(1)

where \( f \) is the kick function (see Ref. 28), \( w_n \) is the rotating argument of periapsis at the \( n \)th revolution, and \( K_n \) is the energy at the \( n \)th revolution. The map is derived by integrating the moon perturbation over one revolution of an unperturbed Keplerian orbit. It is therefore an analytical model of a trajectory of a spacecraft on a near-Keplerian orbit perturbed by a smaller body.

This analytical relationship instead of the full numerical integration of the restricted three-body equations of motion clearly leads to tremendous savings in computational time. Although this map is only an approximation, the qualitative behavior is representative of the full dynamics over many orbital revolutions. It is therefore useful to quickly explore the potential fuel-efficient resonant paths (feasible for near-ballistic trajectories) and find the corresponding starting conditions. Given a number of revolutions \( n \) (found from the desired timescale of the mission) and a particular initial resonant semi-major axis \( a_0 \), the change in energy can be computed as a function of \( w_0 \) by applying the map \( n \) times, i.e., we compute the sequence of pairs \((w_n, K_n)\) which result recursively from a given initial condition \((w_0, K_0)\). The corresponding values of \( w_0 \) that yield maximum energy changes are promising starting conditions for the resonant orbit. An example of this procedure in the Jupiter-Ganymede system is given in Figure 5.

![Figure 5 - \( \Delta a \) versus \( w_0 \) obtained analytically and by integration for \( a_0 = 0.8612 \approx 4:5 \) resonance\(^1\), \( C = 3.0055 \), and 14 inertial revolutions in the Jupiter-Ganymede system](image)

In addition, the analytical two-dimensional map as opposed to full numerical integration of the restricted three-body equations of motion provides other advantages: by plotting the results obtained by the Keplerian Map in phase space \((a \ vs. \ w)\), we can easily visualize a resonant path and understand the dynamical mechanism of the transitions between resonances.\(^2\) For

\(^1\) For numerical integration, the initial semi-major axis at the first apoapsis flyby was selected to be \( a = 0.8538 \) so that the orbital semi-major axis before the flyby was close to be the one of the 4:5 resonance (same principle as for the kick function comparison).
instance, the phase space trajectory of the minimum found above is illustrated as large dots in Figure 6. The background is obtained by applying the Keplerian Map for several initial values \((w, a)\) and following the recursion for thousands of iterates. This phase space reveals the resonance structure which governs transport from one orbit to another. The random scattered points correspond to chaotic motion whereas blank ‘holes’ represent stable resonant islands. For every semi-major axis value \(a_{\text{res}}\) corresponding to a \(K:L\) resonance, there is a band of \(L\) islands. It has been shown that there exists an unstable periodic orbit in the chaotic zone between each island. This observation explains why resonant orbits are so important, they are similar to passes in a chaotic environment, which have to be crossed in order to move in the phase space without getting stuck in stable resonances. This transport mechanism is illustrated in Figure 7. For connecting two distant points, it is therefore necessary to cross a certain number of resonances. For instance, the large dots in Figure 6 give the successive resonant path followed by the minimum found in Figure 5. As expected, the spacecraft jumps around \(w = 0\) between a certain number of resonant bands. This plot therefore provides a graphical way to see how the spacecraft jumps between resonant orbits.

In addition, this plot shows only the resonances that the trajectory may have to traverse, all unnecessary intermediate resonant orbits are automatically skipped. This useful observation is a direct consequence of well-known properties of chaotic Hamiltonian-preserving maps (the Keplerian Map is in this category). The size of the resonance structures can be used to estimate the degree to which resonances are important in a given map. From the plot, we can therefore find visually all the important resonant bands. We will call those required resonances the significant resonant orbits. We acknowledge that this method is not without deficiency as the width of the islands is poorly defined and is dependent on how the map is constructed (initial conditions, number of iterations…). A more rigorous approach would need to rely on other parameters that are better defined and can be evaluated numerically with arbitrary accuracy, like for instance the mean exponential growth factor. However, since we want a simple and quick method, this refinement is beyond the scope of the paper.
possibilities, we are left to a much reduced set of resonances to test. For the Ganymede case, this set is \{6:7, 5:6, 4:5, 7:9, 3:4, 5:7\} (all orbits below 5:7 are not interesting because their semi-major axis is lower than the one of the Ganymede-Europa Hohmann transfer). If one wants a single resonant path only and cannot afford to test multiple cases, the procedure of Figure 5 can be applied to find quickly only one promising (ballistic) resonant path. In our example, the particular ballistic resonant path is 4:5 \rightarrow 7:9 \rightarrow 3:4 \rightarrow 5:7. The time history of the semi-major axis in Figure 8 confirms this result.

As a first step towards implementing our method, we developed a numerical procedure to calculate families of resonance orbits for general values of \(K:L\) and mass ratios in the restricted three body model. The numerical method is briefly described. First, we note that resonant orbits are simply perturbed two body orbits with a specific period and argument of periapsis. The perturbation amplitude is directly related to the distance of the close approach to the smaller body. For resonant orbits with close approach well beyond the Lagrange point, the initial conditions are easy to approximate (simple two-body orbit). We initialize here with a straight forward initial guess and then differentially correct for exact periodicity. The family is then continued by successively targeting different Jacobi constant, \(C\), such that the close approach moves towards the smaller body. A similar continuation technique was implemented by Anderson. Poincare map approaches that seek periodic orbits based on plane crossings are not robust due to the loops associated with the rotating frame (see Figure 12 for example). From iteration to iteration or solution to solution the loops can appear, disappear, or shift causing a discontinuous change in the number of plane crossings. Instead a full dimensioned periodic orbit search is suggested that seeks the full initial conditions and period in order to target periodicity and a desired Jacobi constant. Using the described robust approach, we can pre-compute an exhaustive database of initial solutions of all significant resonant orbits. Provided a resonant path for either the inner or outer moon phase, the initial guess for each leg is obtained by interpolating the initial conditions of the resonant orbit families to the same target Jacobi constant value. In this manner, near-ballistic solutions will arise naturally in the optimization.

Last but not least, we want to point out that using the Keplerian Map is not required in our multiple shooting method, as the resonant path could be obtained by simply enumerating all the possible resonances for a given transfer time. But we include the map based on the insight it provides regarding the dynamics and its ability to facilitate the generation of initial guesses (reduced set of resonant orbits and promising resonant paths). In particular, information about ballistic orbit feasibility is not available by simply enumerating resonant paths.

IV. GENERATION OF UNSTABLE RESONANT ORBITS

Central to the resonant hopping transfer design framework is the use of resonant periodic orbits. In the same way that periodic orbits close to planetary moons provide a basis for mapping and understanding the local dynamics, the resonant periodic orbits that circulate the primary body play a critical role in the global dynamics and efficient transfers within. In the next section, we use the initial conditions from the periodic orbits as initial guesses for the multi-shooting targeting scheme.
Figure 9 and Figure 10 give example data resulting from the resonant periodic orbit finder tool for the 3:4 family at Ganymede. Figure 10 shows important characteristics of the resonant orbits, including stability indices. The second subplot from the top confirms that the orbits are highly unstable. (Note Re|b2|=8.2 on the far right side of plot making the magnitude outside of the stability range of 2 for the entire domain). For details on the stability indices and periodic orbit generation see Ref. 37.

In this section, we describe the systematic procedure of finding fuel efficient inter-moon transfers. In this initial study, impulsive thrusts are used for the control of the trajectory, and we limit ourselves to transfers between two moons only.

As we mentioned in section III, trajectories must go through a complicated phase space structure with coexisting chaotic and regular regions (see Figure 6). In order to have a robust targeting approach in this environment, we combine different state-of-the-art techniques based on chaotic and multi-body dynamical systems theory.

**Forward-Backward Strategy**

Classically, we decompose first the dynamical model into two patched ideal planar restricted three-body problems (PR3BPs), i.e. we split the trajectory in two phases where only one moon at a time affects the motion of the spacecraft. In each phase, the moon of interest is in a prescribed circular orbit about Jupiter within the same plane. In addition to simplifying the model, this allows us to take advantage of the well-known properties of the PR3BPs. Although the approximation is rather crude, it is commonly used for preliminary analysis of space missions. Since we focus on the validation and performance of our multiple shooting method, a refined model is beyond the scope of this paper.

Then, in the phase influenced by the outer moon, we iterate the controlled trajectory forward in time. Likewise, in the inner moon phase, we iterate backward in time. Of course, the boundary points of both phases must be patched together to obtain a full continuous trajectory. This two-fold approach allows a symmetric treatment of each problem. In addition, forward-backward methods have been proven to be very effective for targeting trajectories in chaotic environments. The crucial question of the determination of the boundary condition for the transfer patch point is discussed in the next subsection.

**T-P Graph Targeting**

In our method, a patch point is found using a new graphical tool, the Tisserand-Poincare (T-P) graph, introduced by Campagnola and Russell. On the T-P Graph, level sets of constant Tisserand parameter are plotted in \((r_o, r_p)\) space where the Tisserand parameter is almost equivalent to the Jacobi constant of the PR3BP. During the resonance hopping transfer, the spacecraft moves along the level sets of Tisserand curves. Note that ballistic transfers are fixed in Jacobi constant (and approximately fixed in the Tisserand parameter). The impulsive maneuvers allowed in our model are small enough to not change the Jacobi constant to first order.

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1 This is true because the Poincaré section that generates the T-P graph is far from the minor body.
The intersection point between the Tisserand level sets of two different moons is therefore the target patch point. It is obtained by solving the system:

\[
\begin{align*}
J_{M1} &= \frac{2a_{M1}}{r_a + r_p} + 2 \frac{2r_a r_p}{(r_a + r_p)a_{M1}} \\
J_{M2} &= \frac{2a_{M2}}{r_a + r_p} + 2 \frac{2r_a r_p}{(r_a + r_p)a_{M2}}
\end{align*}
\]

where \(J_{M1}\) is the Jacobi constant of the forward trajectory, \(J_{M2}\) is the Jacobi constant of the backward trajectory, \(a_{M1}\) is the semi-major axis of the first moon and \(a_{M2}\) is the semi-major axis of the second moon. We call the pair \((r_a^*, r_p^*)\) the solution of this system.

It follows that the T-P graph provides a simple way to calculate the patch point of the two parts of the trajectory. The forward phase targets \(r_p^*\) and the backward phase targets \(r_a^*\). Therefore, the problem is reduced to a one-dimensional targeting problem and the solution to the forward and backward problems are uncoupled (to first order assuming the impulsive maneuvers do not change the respective Jacobi constants). We are therefore able to break the original problem at the patching point into two sub-problems, and each sub-problem can be independently optimized (opening the possibility of parallel computation). This approach is significantly easier than previous methods. Traditionally, one must target the coupled six-states of an arbitrary non-optimal, switching orbit (the Hohmann orbit in general). An approach is significantly easier than previous methods.

Multiple Shooting Optimization

The multi-body system is known to be very unstable and chaotic, which results in a very high sensitivity with respect to initial conditions and small control perturbations. In those conditions, optimizing a trajectory is therefore difficult to achieve, even with the forward-backward strategy. The multiple shooting method attempts to limit the sensitivity issue by splitting the integration interval to reduce error propagation. Additional matching constraints are then imposed to get a continuous solution on the whole interval. This strategy is generally found to be more efficient and robust.

In addition, as suggested in section 3, the concept behind multiple shooting is in good agreement with targeting theory in chaotic dynamical systems. Previous authors mentioned that forward-backward direct targeting can yield poor results and is thus not enough when the resonant structure of the problem is complex. The three-body gravity-assist problem falls exactly into that category as trajectories can get trapped in the multiple resonant bands shown in Figure 6 and Figure 7. This issue can be overcome by finding the unstable resonant periodic orbits that lie in the chaotic passes of resonant bands (from the algorithm of section 4). Those orbits are then used as starting points for the intermediate nodes of multiple shooting. This way, the resonant path of the controlled trajectory is preselected, and the solution is therefore encouraged to fall into the pass regions which lead to the desired resonance transport. In other words, the multiple shooting concept comes naturally from the understanding of the chaotic phase space structure of the problem. It is therefore expected to be efficient in overcoming the sensitivity of chaotic motion.

The phases of multiple shooting are illustrated in the rotating frames of each moon in Figure 12. As explained above, the nodes are located at each flyby to increase robustness and to allow the easy use of resonant periodic orbits as initial guesses. The transfer times between the nodes are free. Controlling the trajectory is obtained through small impulsive maneuvers that are optimized by the solver. As shown in Figure 1, a resonant gravity-assist is in general characterized by only one mid-course maneuver located on the other side of the trajectory with respect to the moon. For more controllability, we introduce additional maneuvers at each periapsis (resp. apoapsis) of the outer moon (resp. inner moon) phases, as well as at the flyby itself. Note that we are aware that a fixed location of the maneuver is not optimal, and in the spirit of Olympio’s work, a better approach would be to optimize the number and location of the maneuvers as well. However, from classical orbital mechanics, we know that an efficient way of changing the orbital elements at one apse is to use an increment of velocity provided at the opposite apse, so we can expect our choice to be near optimal.
In each PR3BP phase, the multiple shooting optimization problem is formulated as a nonlinear parameter optimization sub-problem, where the control variables are the initial positions, magnitude and direction of the maneuvers, and times of all the closest approaches. It follows that the decision vector of the unknowns for the $i$th leg is:

$$
\begin{bmatrix}
X_{0,i} \\
X_{f,i} \\
\Delta V_{1,i} \\
\vdots \\
\Delta V_{n,i}
\end{bmatrix}
$$

(3)

where $n$ is the total number of DSMs for the corresponding leg.

The solver must minimize the total $\Delta V$ needed while fulfilling the constraints given by:

$$
G = \begin{bmatrix}
X_{prop}^1 - X_{node}^2 \\
\vdots \\
X_{prop}^{n-1} - X_{node}^n \\
r_{p/a} - r_{p/a}^*
\end{bmatrix}
$$

(4)

where $X_{prop}$ is the state at one match point coming from the propagated state at the previous node, $X_{node}$ is the current state at one match point, $r_{p/a}^*$ is the final apse value of the current trajectory, and $r_{p/a}^*$ is the targeted apse for the end of the transfer. The last constraint is the $r_a$ (resp. $r_p$) targeting constraint while the other constraints express the continuity at each multiple shooting node.

In this hyper-sensitive problem, an accurate jacobian of the constraints is necessary to increase the robustness of the approach. The jacobian has the following sparse form:

$$
Jac = \begin{bmatrix}
-I & 0 & 0 & 0 \\
\Phi_i & -I & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & dr_{a/p}^* dxf \cdot \Phi_n
\end{bmatrix}
$$

(6)

where $\Phi_i$ is the state transition matrix of the $i$th multiple shooting segment (including state and time components). The state transition matrices are determined very accurately using the complex variable method described in Ref. 41. We note that the complex method is underutilized in the astrodynamics community and new applications and extensions are a promising area of future work.
All in all, the sub-problems to solve can be formulated as follows:

$$\min_{Z_1,\ldots Z_n} \sum_{i} \sum_{j=1}^{n_i} \| \Delta V_{j,i} \|$$

$$s.t. \quad g(Z_1,\ldots, Z_n) = 0$$

A first guess is generated using resonant paths and resonant periodic orbits (at appropriate energy levels) obtained with the methods described previously. The times and states of each node are therefore specified. Thrust impulses are initialized to zero. The solver chosen for this study is SNOPT, a state-of-the-art, general purpose, nonlinear solver for large optimization problems. The implementation is performed in Fortran 90.

**End-to-End Optimization**

Ideally, the forward and backward parts of the trajectory should patch perfectly, without the need to refine the trajectory. However, in practice, an end-to-end optimization step is required to obtain perfect continuity. One reason is that the computations of $r_a$ and $r_p$ are approximate as they rely on two-body theory, so an error is introduced in the calculation of the patching point. In addition, even if the thrust impulses are small, they generally modify the Jacobi constant and this small change should formally be reflected when the system of Eq. 2 is solved.

For achieving exact continuity, it is necessary to adjust the phase between the inner moon and the outer moon at the time of intersection. The backward and forward algorithms of the previous subsection are then combined to optimize the whole trajectory. Future work can include in this step more refined ephemeris models as well.

**Summary**

The overall algorithm is briefly summarized now. By construction, the algorithm is very robust and should converge for any resonant path (according to intuition and preliminary numerical experiments).

**Step 0. Initialization**

Parameters of the problems (gravitational parameters of the bodies, distances between the bodies) are given. The estimated flight times, the Jacobi constants, the initial and final resonance orbits are also required.

**Step 1. Selection of the resonant path**

From the Keplerian Map or any other method, a resonant path for both moons is selected.

**Step 2. Generation of the initial guess**

The initial guess is built by stacking states at flybys of the unstable resonant orbits corresponding to the resonant path and selected Jacobi constant. We could also use results produced by two-body Vinfinity-leveraging techniques. Moreover, we noted in section 3 that Keplerian Map iterations may match reasonably well with the full dynamics (see Figure 5). Therefore another strategy can be the direct use of the states of the Keplerian Map as an initial guess.

**Step 3. Uncoupled T-P Graph targeting**

The intersection of the T-P Graph is targeted backward and forward using a multiple shooting method.

**Step 4. End-to-End optimization**

Based on the times associated with of the results of step 3, the initial phasing between the moons is selected to ensure patching of the systems at the switching orbit. Finally, the uncoupled solutions are used as an initial guess for an end-to-end optimization for the entire inter-moon transfer.

**VI. NUMERICAL RESULTS**

In this section, we demonstrate the efficiency of our method by computing optimal end-to-end trajectories from a resonance close to Ganymede to a resonance close to Europa. This is a common benchmark problem studied by many authors. Since our procedure is systematic, we can perform a rudimentary $AV$ vs flight time trade study to test a variety of optimized resonant paths. We select different combinations of the significant resonant orbits given by the Keplerian Map (see section III) for Ganymede and Europa. The Jacobi constants of the two portions of the trajectories are initially set to $J_{Ganymede} = 3.0068$ and $J_{Europa} = 3.0024$. Those values are known, from previous numerical experiments, to lead to feasible low-energy transfers. Furthermore, we seek energy levels that are consistent with low-energy captures or escapes at the respective Moons. The minimum energy level (maximum C) possible for escape or capture is of course the energy level when Hill’s neck emerges as part of the zero velocity curves that separate valid and forbidden regions. The initial and final resonances, 4:5 and 6:5, respectively, are chosen because they can be reached by simply ‘falling off’ Halo orbits close the moons.

The scatter plot of the results is shown in Figure 13 along with an approximate Pareto Front. Table 1 details each resonant hopping sequence of this plot. The last resonant orbits of the resonant path of each phase (5:7...
and 7:5 respectively) are only used as a guess of the patch point. This is possible because the switching orbit occurs close to 5:7 and 7:5 (see Figure 19), but this is only a coincidence due to the orbital characteristics of the Ganymede and Europa transfer (it is generally not \(a:b\) then \(b:a\)).

Despite the high sensitivity of the problem, convergence is almost always achieved, and the average computational time for each case is in the order of two or three minutes using the Intel Fortran compiler and a 2.0 GHz processor (one minute per phase approximately). Generating this set of solutions therefore demonstrates that our approach is systematic, fast, and robust.

The theoretical minimum \(\Delta V\) from Vinfinity leveraging can be computed from a quadrature.\(^{16}\) Using this equation, the minimum \(\Delta V\) for a 4:5-to-6:5 transfer is found to be 183 m/s. We can see that our method gives far lower \(\Delta V\). On our best transfer (55 m/s), we get a 70 % reduction in \(\Delta V\) compared to the best theoretical \(\Delta V\) possible from VILM. In addition, comparison of our results with those of a recent detailed study of VILM transfers\(^{33}\) shows that our flight times are at the same order of magnitude.

These results suggest that the resonant paths from Ref. 33 are good initial paths to examine. However, according to Ref. 14 and conventional wisdom, if the exact Vinfinity leveraging (high-energy) solution is used as an initial guess for an optimizer, then a nearby local minimum in the higher fidelity model will be found with similar results. Instead, if a robust solver is used in conjunction with the periodic resonant orbits as an initial guess, then the low energy, low \(\Delta V\) alternative solution can be found.

It is clear that solution 3 can be seen as a good compromise between fuel consumption and time. For this transfer, there are two Ganymede flybys and four Europa flybys. A total \(\Delta V\) cost of 59.5 m/s is required and the total flight time is 158.5 days, which is well within conceivable mission constraints. As a basis of comparison, it takes up to 5 m/s just to navigate a flyby\(^{44}\), so the \(\Delta V\) cost is almost at the level of statistical maneuvers. The corresponding entire trajectory of solution 3 is shown with time histories of semi-major axis and apse distances in Figure 14 - Figure 17.

The data of this example are given in Appendix 3. In particular, from Figure 17, we see that the semi-major axis is decreased sequentially, as expected. First, the trajectory gets its \(r_p\) reduced with two flybys of
Ganymede. Then, the spacecraft passes naturally to the control of Europa and accordingly reduces its $r_a$. Ref. 21 gives a high altitude closed periodic orbit at Europa for a Jacobi constant of 3.0023 (ID 1486948), therefore our final resonance obtains an energy value, $J=3.0024$, that is consistent with loose capture around Europa. We emphasize that the trajectory does include phasing and several fully integrated flybys of both Ganymede and Europa.

Further insight of the dynamics is seen when plotting the spacecraft trajectory on the TP-graph (see Figure 18 and Figure 19). The spacecraft begins its transfer around the center of the figure on a low-energy Ganymede Tisserand curve. The spacecraft has its $r_p$ reduced via Ganymede gravity assists until it reaches the intersection with the desired Europa Tisserand level set. Then the spacecraft falls under Europa’s influence where its $r_a$ is decreased while its $r_p$ is approximately kept constant (according to the level Tisserand curve). In Figure 19, we can verify that the optimized switch point is very close to the theoretical switching point predicted by the TP-graph theory. The 5:7 and 7:5 resonances of the initial Jacobi constants are also shown to point out that the switch indeed occurs in their neighborhood, as expected.
VII. FUTURE WORK

Additional work is needed to fully test and validate the properties of the algorithm across a wide spectrum of problems. Other problems of interest include Callisto-Ganymede transfers, Saturn or Uranus satellite tours, etc. For the moment, the algorithm is limited to two moons only, but we intend to extend it to specify intermediate moons as well.

In addition, we acknowledge that the scope of our methodology is presently limited since we must start and arrive at some arbitrary resonances with respect to the moons. In practice, it is necessary to connect the inter-moon portion of the trajectory with carefully selected science orbits around the moons, but this step remains an open issue. A promising method is to combine resonance hopping with the manifold dynamics which control the transport near the libration points of the moons.\textsuperscript{21,23}

Moreover, another capability that is useful to implement for the design of a real mission is the consideration of a full ephemeris model. The actual implementation would be quite straightforward since we are already performing the integrations in the inertial frame with a simple planar, circular ephemeris. It would be interesting to check if the properties of the dynamical transport structure of section III are generally preserved from the influence of perturbations. Also, we intend to elaborate a methodology to take advantage of the so-called Laplace resonances that keep the orbital periods of Io, Europa, and Ganymede in a ratio of 1:2:4. By adjusting the phasing (and therefore the epoch) properly, ‘extra kicks’ are available by the moons.

Finally, it would be interesting to design a low-thrust trajectory for inter-moon transfers. Even if the current ESJM baseline mission does not plan to use low-thrust propulsion, this is not an option to be overlooked. The canceled JIMO mission included an ion engine for performing a Jupiter tour,\textsuperscript{45} and there will be other outer planet missions in the future that might reconsider low thrust. We believe that the method outlined here can be readily modified to incorporate low thrust using the impulsive parameterization of Sims and Flanagan.\textsuperscript{46} Also, since low-thrust trajectories are much more challenging to optimize due to the very high number of control variables, a good initial guess is vital. The work of this paper could serve as a first guess generator since low-thrust optimal solutions have been empirically determined to follow resonant periodic orbits.\textsuperscript{47}

CONCLUSION

In this paper, we initiate a new systematic, fast and robust multi-shooting method to generate low-energy inter-moon trajectories with multiple resonant gravity assists (‘resonance hopping’). Using insight from dynamical systems theory, we can quickly generate promising approximate trajectories and find good resonant paths through which the spacecraft must go in order to efficiently perform the transfer. The corresponding unstable resonant orbits are computed and used as an initial guess that is fed to the optimization algorithm. Note that the resonant periodic orbit generator described here is available by request. By seeking the intersection between the two backward and forward transit trajectories on the T-P Graph (a new graphical tool of the three-body system), transfers between moons are possible with thrust impulses much smaller than that necessary for $V_{\infty}$ leveraging transfers. With a simple non-exhaustive search, we produced families of Fuel-Time Pareto optimized trajectory solutions between Ganymede and Europa. In particular, we were able to design a transfer which required only 59 m/s and 158 days.

In addition, a by-product of this work is a deeper understanding of the dynamic structure of resonance passes in the three body problem. We introduced the concept of significant resonant transitions and explained why an efficient trajectory is likely to cross them.

The multi-body resonant hopping technique is demonstrated as a promising and advantageous alternative to the conventional patched conic methods. Overall, this work can be seen as the next step in the direction towards the automated design of satellite tours using multi-body dynamics.

ACKNOWLEDGEMENTS

The authors thank Johnny Kwok, Jon Sims, Anastassios Petropoulos, and Thierry Dargent for support of the project. The authors thank Shane Ross for sharing the data of his multi-moon orbiter trajectory. This work was sponsored in part by Thales Alenia Space and the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

REFERENCES


APPENDIX 1

At apoapsis of the trajectory, the kick function is given by:

\[ f = -\frac{2}{\sqrt{p}} \int_0^{2\pi} \left( \frac{r(v)}{r_2(v)} \right)^3 \sin(\theta(v)) - \sin(\theta(v)) \right) dv \]

where:

\[ r = \frac{a(1-e^2)}{1 + e \cos \nu}, \quad r_2 = \sqrt{1 + r^2 - 2r \cos \vartheta} \]

\[ E = \text{sign}(\nu) \left[ \arccos \left( \frac{e + \cos \nu}{1 + e \cos \nu} \right) \right] \]

\[ t = a^{3/2} \left( E - e \sin E - \pi \right), \quad \vartheta = w - \pi + \nu - t \]

Notations consistent with Ref. 28 are adopted and we assume \( \mu = 1 \). Here \( \nu \) is the true anomaly of the trajectory.

APPENDIX 2

Let \( w, a, C \) (Jacobi constant) given. Assume that the flyby is at periapsis (modifying the equations for the apoapsis case is straightforward). We want to deduce the corresponding state in rotating frame. First we compute the eccentricity from the well-known expression of the Jacobi constant in function of \( a \) and \( e \):

\[ e = \sqrt{1 - \frac{1}{4a} (C - \frac{1}{a})^2} \]

Since we are at periapsis at an angle \( w \) with respect to the secondary body, the position is:

\[ R = \begin{bmatrix} \cos w & -\sin w & r_p \\ \sin w & \cos w & 0 \end{bmatrix} \]

where \( r_p = a(1-e) \)

From the energy equation, the norm of the inertial velocity is:

\[ v = \sqrt{2 \left( \frac{1}{r_p} - \frac{1}{2a} \right)} \]

After rotating the velocity and going from inertial to rotating coordinates, we get:

\[ V = \begin{bmatrix} \cos w & -\sin w & v \\ \sin w & \cos w & 0 \end{bmatrix} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} R \]

APPENDIX 3

We give here the solution vectors and the numerical data of the example given in section VI:

\[ \mu_{\text{Ganymede}} = 7.802 \times 10^{-5} \]

\[ d_{\text{Ganymede}} = 1070.4 \text{ km (orbital radius of Ganymede)} \]

Initial inertial angle of Ganymede: \( 0^\circ \)

\[ \mu_{\text{Europa}} = 2.523 \times 10^{-5} \]

\[ d_{\text{Europa}} = 670.9 \text{ km (orbital radius of Europa)} \]

Initial inertial angle of Europa: \( 319.5^\circ \)

The scaling is given by:

- position: \( 1070.4 \text{ km} \)
- velocity: \( 10.880850460047206 \text{ km/s} \)
- impulse: \( 1 \text{ m/s} \)
- time: \( 27.326295350266523 \text{ h} \)

In order to preserve the dynamics of the PRTBP, we note that the origin is the barycenter of the Jupiter-moon system and therefore Jupiter instantaneously changes positions (albeit only on the order of \( \sim 100 \text{ km} \)) at the time of the switching orbit.

In the following, we give the solution vector for each leg in the same format as Eq. 3, and a mesh vector corresponding on the time of the nodes of each leg.

\[ t_{\text{node}}(j) = t_0 + K_{\text{mesh}}(j) \times (t_f - t_0) \]

Ganymede portion of the trajectory:

Leg 1:

\[ K_{\text{mesh},1} = [0.0, 0.1, 0.3, 0.5, 0.7, 0.9, 1.0] \]

\[ X_{\text{Leg},1} = \begin{bmatrix} 0.971772553135740 \\ 2.000000000000000E-003 \\ -1.99999999940707E-003 \\ 0.939663097665845 \\ 0.200000000000000 \\ 26.1383793256402 \\ 1.902195705689637E-002 \\ 1.000000000000000E-007 \\ 1.000000000000000E-007 \\ -2.46632086189342 \\ 1.04434783719898 \\ -2.30313426639315 \\ 1.49055001006334 \\ -0.2070215436293 \\ 1.81924489812533 \\ -1.71199873686942 \\ 2.0056226946983 \\ -1.28461657105624 \end{bmatrix} \]

Leg 2:

\[ K_{\text{mesh},2} = [0.0, 0.125, 0.375, 0.625, 0.875, 1.0] \]

\[ X_{\text{Leg},2} = \begin{bmatrix} 0.971772553135740 \\ 2.000000000000000E-003 \\ -1.99999999940707E-003 \\ 0.939663097665845 \\ 0.200000000000000 \\ 26.1383793256402 \\ 1.902195705689637E-002 \\ 1.000000000000000E-007 \\ 1.000000000000000E-007 \\ -2.46632086189342 \\ 1.04434783719898 \\ -2.30313426639315 \\ 1.49055001006334 \\ -0.2070215436293 \\ 1.81924489812533 \\ -1.71199873686942 \\ 2.0056226946983 \\ -1.28461657105624 \end{bmatrix} \]

Europa portion of the trajectory:

Leg 1:

\[ K_{\text{mesh},1} = [0.0, 0.1, 0.3, 0.5, 0.7, 0.9, 1.0] \]

\[ X_{\text{Leg},1} = \begin{bmatrix} 0.971772553135740 \\ 2.000000000000000E-003 \\ -1.99999999940707E-003 \\ 0.939663097665845 \\ 0.200000000000000 \\ 26.1383793256402 \\ 1.902195705689637E-002 \\ 1.000000000000000E-007 \\ 1.000000000000000E-007 \\ -2.46632086189342 \\ 1.04434783719898 \\ -2.30313426639315 \\ 1.49055001006334 \\ -0.2070215436293 \\ 1.81924489812533 \\ -1.71199873686942 \\ 2.0056226946983 \\ -1.28461657105624 \end{bmatrix} \]

Leg 2:

\[ K_{\text{mesh},2} = [0.0, 0.125, 0.375, 0.625, 0.875, 1.0] \]

\[ X_{\text{Leg},2} = \begin{bmatrix} 0.971772553135740 \\ 2.000000000000000E-003 \\ -1.99999999940707E-003 \\ 0.939663097665845 \\ 0.200000000000000 \\ 26.1383793256402 \\ 1.902195705689637E-002 \\ 1.000000000000000E-007 \\ 1.000000000000000E-007 \\ -2.46632086189342 \\ 1.04434783719898 \\ -2.30313426639315 \\ 1.49055001006334 \\ -0.2070215436293 \\ 1.81924489812533 \\ -1.71199873686942 \\ 2.0056226946983 \\ -1.28461657105624 \end{bmatrix} \]
<table>
<thead>
<tr>
<th>Leg 1:</th>
<th>$K_{\text{mesh,1}} = [0.0, 0.1, 0.3, 0.5, 0.7, 0.9, 1.0]$</th>
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<tr>
<td>$X_{\text{Leg,1}} = [0.6377817867856313, 8.245934717864093E-004, 1.83741622748608E-003, 1.330738079297167, -7.43753866170456E-002, 0.917102399680224, 44.9895132805651, 47.2652495497626, 1.144997086150060, -0.961187745920894, -0.56450382922955398]</td>
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Europa portion of the trajectory (backward):

<table>
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<th>Leg 2:</th>
<th>$K_{\text{mesh,2}} = [0.0, 0.07142, 0.214285, 0.357142, 0.5, 0.642857, 0.785714, 0.928571, 1.0]$</th>
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<td>$X_{\text{Leg,2}} = [0.6389058055538506, 1.071722538262295E-002, -1.88614618665743E-002, 1.32393470400369, -18.997208288828, 1.7156136470926, 0.173781744405468, 4.492524679035975E-002, 1.1985028950608, 0.127591849099997, 1.18393242448054, 0.121502952758578, 1.17106824105797, 0.113850808596218, 1.15637887534257, 0.105379173253908, 1.14230615714532, 9.663731200789621E-002, 1.12674956780231, 8.5013925058050E-002, 1.11213415730840, 7.277346010931435E-002]</td>
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<tr>
<th>Leg 3:</th>
<th>$K_{\text{mesh,3}} = [0.0, 0.1667, 0.5, 0.8333, 1.0]$</th>
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<tr>
<th>Leg 4:</th>
<th>$K_{\text{mesh,4}} = [0.0, 0.0625, 0.1875, 0.3125, 0.4375, 0.5625, 0.6875, 0.8125, 0.9375, 1.0]$</th>
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<tbody>
<tr>
<td>$X_{\text{Leg,4}} = [0.6411721015792678, 6.641401042273383E-003, -1.723152201970365E-002, 1.35273226251703, -59.8095937939935, -94.3377876036802, 0.11102285329652, 0.236850511889786, 2.37110565087934, -1.02108134048572, 2.3386421693169, -1.0060687001643, 2.20241383429252, -0.971572032312953, 2.07396907201318, -0.933226713085202, 1.937333388777707, -0.8921836444324225, 1.81956550509991, -0.85000540395323, 1.70156042962288, -0.810393935764708, 1.58335267417909, -0.769129910933763]</td>
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<table>
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<tr>
<th>Leg 5:</th>
<th>$K_{\text{mesh,5}} = [0.0, 1.0]$</th>
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<td>$X_{\text{Leg,5}} = [0.6458569137580, 5.3809897661587E-003, -8.017168095321E-002, 1.35561768142, -94.33778860368, -96.46185637964, 1.26758151350528, -1.07624325586504]</td>
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