ELECTRIC PROPULSION OPTIONS FOR A PROBE TO EUROPA

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ABSTRACT :

In this paper an option for a mission to Europa has been investigated taking into account electric propulsion as main source of thrust. A direct optimisation technique has been employed to design an optimal trajectory maximising the payload mass to Europa. Previous to that a global search for potential optimal solutions has been performed. In particular, since the trajectory involve a large number of swing-byes, of both inner planets and jovian moons, a special procedure has been developed to study this kind of trajectories. After proving the effectiveness of the global search an optimal trajectory is presented demonstrating how a fully electric propulsion option even to distant planets could be attractive.

1 - INTRODUCTION

Europa, one of the moons of Jupiter, is of course one of the most interesting target in the solar system for exobiology studies. In fact the presence of water, probably liquid under the superficial layer of ice, suggests the possibility of prebiotic life. A mission aimed to the exploration of this peculiar moon would be extremely interesting but, at the same time, quite challenging due to the distance from Earth and from the Sun. In fact such a mission would be extremely demanding in terms of communications, power and Δv requirements. All the missions to Jupiter or to the outer part of the solar system, such as Galileo or Cassini, have employed chemical propulsion as main propulsion system and RTG technology to generate the required power on-board. However an alternative, not yet explored, would be to use electric propulsion or a combination of electric and chemical propulsion trying to exploit at best the two.

In this paper different options, that use electric propulsion as main source of thrust to reach Europa, are investigated. The requirements in terms of Δv are reduced resorting to multiple Gravity Assist(GA) manoeuvres both of the inner planets, Venus, Earth and Mars, and of the jovian moons. In addition, in order to reduce the demands in terms of operating time of the engine, the gravity attraction of Juppiter has been taken into account. The main concern of this study is to asses the actual feasibility of a trip to Europa using electric propulsion as main source of thrust and to outline the related issues and requirements.

Now the design of a transfer trajectory combining SEP and GA can be regarded as a general trajectory optimisation problem. The dynamics of the spacecraft is governed mainly by the gravity attraction of the Sun, when the spacecraft is outside the sphere of influence of a planet, and by the gravity attraction of the planet during a gravity assist manoeuvre. Low-thrust propulsion is then used to shape trajectory arcs between two subsequent encounters and to meet the best incoming conditions for a swing-by. Therefore, in this paper a direct optimisation approach[1] has been used to design an optimal trajectory to Europa exploiting several swing-bys of Venus, Mars, Earth and of the jovian moons. In order to take into account swing-bys, the trajectory has been split into several phases, each phase corresponding to a trajectory arc connecting two planets. On each phase a Finite Elements in Time technique[2] has been used to transcribe differential equations, governing the dynamics of the spacecraft, into a set of algebraic nonlinear equations.

Prior to the optimisation of the trajectory a particular technique has been used to generate a first guess solution involving different possible combinations of swing-bys with inner planets and with jovian moons. The proposed technique, which seeks for possible combination of swing-bys, minimising the requirements in terms of Δv, within a given range of possible launch dates has provided different interesting scenarios for a transfer orbit to Eauropa. In this paper some of the most interesting opportunities found are presented demonstrating the effectiveness of the proposed approach.

2 - PROBLEM FORMULATION

A transfer to Europa using solar electric propulsion as main source of thrust requires involves a considerable number of gravity assist manoeuvre to reduce the requirement in terms of $\Delta\mathbf{v}$, in particular when the spacecraft reaches the jovian system. Therefore two principal attracting bodies have been considered: the Sun during cruise from Earth to Jupiter, using Jupiter as disturbing body, and Jupiter, during the tour of jovian moons, considering the Sun as perturbing body. In addition the dependency of low-thrust on the power provided by the solar panels is taken into account. Since two different major reference systems are considered and a huge number of swing-bys are required a multiphase approach has been used with two different dynamic models for the tweo reference systems and an appropriate set of interphase constraints to model swing-bys. Then all the phases are assembled together, forming a single NLP problem which has been solved efficiently by a sparse sequential quadratic programming algorithm.

2.1 - DYNAMIC MODEL

A spacecraft is modelled as a point mass subject to the gravity attraction either of the Sun, during the cruise from the Earth to Jupiter, or of Jupiter, during capture and the tour of jovian satellites, and to the thrust provided by one or more low-thrust engines. The motion of the spacecraft is described in the J2000 mean ecliptic reference frame centred in the Sun (Figure 1)., during cruise, and in the J2000 mean equatorial reference frame centered in Jupiter The three components of the thrust vector **u** represent the control:

$$\begin{aligned}\dot{\mathbf{r}} &= \mathbf{v}\\ \dot{\mathbf{v}} &= \nabla U(\mathbf{r}) + \nabla U_B(\mathbf{r}) + \frac{\mathbf{u}}{m}\end{aligned} \qquad \textbf{[1.]}$$

$$\dot{m} = -\frac{u}{I_{sp} g_0} \qquad \textbf{[2.]}$$

where the gravity potential of the principal attracting body, either the Sun or Jupiter, with gravity constant μ, is a function of the position vector $\mathbf{r} = \{r_x, r_y, r_z\}^T$:

$$U(\mathbf{r}) = \frac{\mu}{|\mathbf{r}|} \qquad [3.]$$

and the disturbing potential due to the gravity of a third body is:

$$U_B(\mathbf{r}) = \mu_B \left(\frac{1}{\mathbf{d}} - \frac{\langle \mathbf{d}, \rho \rangle}{\rho^3} \right) \qquad [4.]$$

where $\boldsymbol{\rho}$ is the position vector of the perturbing body with respect to the principal one, $\mathbf{d}=\mathbf{r}-\boldsymbol{\rho}$ is the position vector of the spacecraft with respect to the perturbing body and μ_B is the gravity constant of the perturbing body. The state and the control vectors are then defined as follows:

$$\mathbf{x} = \{r_x, r_y, r_z, v_x, v_y, v_z, m\}^T; \qquad \mathbf{u} = \{u_x, u_y, u_z\}^T; \qquad [5.]$$

where m is the mass of the spacecraft. An upper bound T_{max} and a lower bound T_{min} was put on the thrust magnitude:

$$T_{\min} \le u = \sqrt{u_x^2 + u_y^2 + u_z^2} \le T_{\max} \qquad [6.]$$

The upper bound is the maximum level of thrust provided by the selected low-thrust engine, the lower was taken $1\text{x}10^{-4}$ times T_{max} to avoid singularities in the Hessian matrix when minimum mass problems are solved. I_{sp} is the specific impulse of the engine and g_0 the gravity constant on Earth surface.

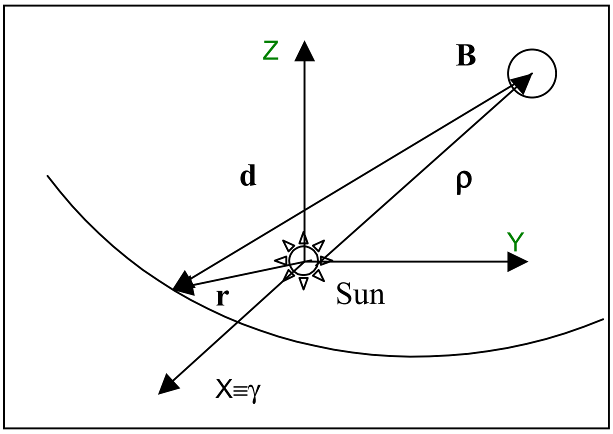


Figure 1 - Inertial reference frame centred in the Sun: the xy plane is the ecliptic plane and x axis points toward the 2000 mean vernal equinox.

2.2 - Swing-by Model

The simplest way to model a gravity assist manoeuvre is to resort to link-conic approximation: the sphere of influence of a planet is assumed to have zero radius and the gravity manoeuvre is considered instantaneous. Therefore the instantaneous position vector is not affected by the swing-by:

$$\mathbf{r}_i = \mathbf{r}_o = \mathbf{r}_P \qquad [7.]$$

where $\mathbf{r}_i$ is the incoming heliocentric position, $\mathbf{r}_o$ is the outgoing heliocentric position vector and $\mathbf{r}_p$ is the planet position vector, all taken at the epoch of the encounter. For an ideal hyperbolic orbit, not subject to perturbations or Δv manoeuvres, the modulus of the incoming relative velocity must be equal to the modulus of the outgoing relative velocity:

$$\tilde{v}_i = \tilde{v}_o \qquad \textbf{[8.]}$$

Furthermore the outgoing relative velocity vector is rotated, due to gravity, of an angle $\beta = \pi - 2\delta$ with respect to the incoming velocity vector and therefore the following relation must hold:

$$\tilde{\mathbf{v}}_o^T \tilde{\mathbf{v}}_i = -\cos(2\delta)\tilde{v}_i^2 \qquad \textbf{[9.]}$$

where the angle of rotation of the velocity is defined as:

$$\delta = \mathrm{a}\cos\left(\frac{\mu}{\tilde{v}_i^2 \tilde{r}_p + \mu}\right) \qquad \textbf{[10.]}$$

All quantities with a tilde are relative to the swing-by planet and $\tilde{r}_p$ is the periapsis radius of the swing-by hyperbola.

2.3 - POWER MODEL

Since one of the major issues for an interplanetary transfer using solar electric propulsion is the power available to the engine, the dependency of the thrust modulus on the power provided by the solar arrays has been taken into account. The maximum thrust that the engine yields, is determined taking into account the specific thrust F_{sp} the effective input power P_{in} provided by the power system and an efficiency coefficient η_e:

$$F_{\max} = \eta_e P_{in} F_{sp} \qquad \textbf{[11.]}$$

The effective input power is given by the effective power produced by the solar arrays minus the power required by the spacecraft P_{ss}:

$$P^*_{in} = P_{eff} - P_{SS} \qquad \textbf{[12.]}$$

In order to take into account the degradation of the solar arrays due to temperature and the reduced power due to the increasing distance from the sun, the power provided by the solar arrays during the transfer trajectory is here expressed as:

$$P_{eff} = \eta_S \frac{P_{1AU}}{R_S^{\ 2}}\left[1 - C_T(T_S - T_0)\right]\cos\alpha \qquad \textbf{[13.]}$$

where P_{1AU} is the power at one Astronomical Unit, Ts is the temperature of solar arrays, R_S is the distance from the Sun, T_0 the reference temperature, C_T is the temperature coefficient which express the reduced performance of the panel with temperature increase, η_S is a coefficient to account for all other degradations sources and α is the solar array sun aspect angle, i.e. the angle between the normal to the cell surface and the sun direction. The steady state surface temperature of the solar panels is here taken as function of the distance from the sun:

$$T_S = \left[\frac{S_0 \alpha_s \cos\alpha}{R_s^2 \sigma \kappa \varepsilon}\right]^{0.25} \qquad \textbf{[14.]}$$

where S0 is the solar constant at 1 AU, σ is the Stefan-Boltzmann constant, αs is the surface absorbivity is the solar spectrum ad ε is the surface emissivity is the infrared spectrum, κ is a coefficient which takes into account the surface area radiating in the infrared spectrum, with respect to the one that receives the solar input. A maximum power that can be handled by the PPU is assumed to represent the upper limit for the engine thrust.

$$P_{in} = \min(P_{in}^*, P_{\max}) \qquad \textbf{[15.]}$$

The required power is dimensioning for the design of the solar arrays and power system and therefore it provides estimation for the overall dry mass of the spacecraft. Power supply characteristics are summarised in table 1.

Table 1. Power system characteristics

PARAMETER	η_e	η_S	C_T	T_0	κ	ε	α	T_{max}	P_{SS}
value	0.9	0.9	3d-4K^{-1}	290K	1.8	1.0	0.8	423K	300W

3 - OPTIMISATION APPROACH

The design of an optimal trajectory involving a huge number of swing-bys with multiple resonant orbits has been approached in two steps. First of all, procedures to find a set of first guess solutions (FGS) have been developed, thereby solving a global optimisation problem. Afterwards, the most promising first guesses were optimised with a direct optimisation method. In particular a procedure has been developed for tour design yielding an accurate estimate of the number of swing-bys, orbit periods and encounter dates. In addition a chemical option has been investigated for comparison using a direct optimisation approach.

3.1 - GLOBAL SEARCH

The issue of the global search problem is to generate an optimal solution looking in the domain of possible paths to mission destination. In order to achieve this challenging task, simplified physical models were adopted and particular assumptions were made on trajectory type, thus decreasing the number of possible alternatives and allowing the implementation of simple and fast optimisers. In particular, coplanar circular orbits were assumed for planets. Since different strategies were assumed for interplanetary transfer and for Jovian system tour, two optimising procedure have been developed on purpose.

3.1.1 - *Interplanetary Transfers*

In the Solar system, high distances and long revolution periods of planets suggest to look for simple but highly efficient strategies, made up of Hohmann-transfer legs or phases, in order to achieve the energy requirements within a transfer time as short as possible. The main problem is to find the best configuration of planets that minimise the Δv required to reach a given target. Since multiple swing-bys are used to reduce the requirements in terms of Δv the problem turns out to be finding the best configurations of planets that allows to go from one gravity manoeuvre to the other minimising midcourse corrections. The basic hypothesis is that a sequence of minimum energy leg (Hohmann transfers) linking subsequent encounters, or swing-bys, can represent a good initial guess giving a good estimate of the minimum cost to actually perform the transfer.

In this frame, planet encounters (i.e., boundary condition for each leg) occur only at apsidal points of Hohman arcs. Moreover, no $\Delta\mathbf{v}_{GA}$ model was implemented, since it has been assumed that each GA manoeuvre can provide the required minimum $\Delta\mathbf{v}$ to go from one planet to the other. Therefore, sequences that require energetic gap that are too high compared to the maximum $\Delta\mathbf{v}$ a GA manoeuvre can provide, are not included in the search. Furthermore, discontinuities in time were admitted between two consecutive phases and the optimisation problem becomes to find not just the best sequence but the most convenient set of launch and encounter dates that minimises the temporal gap between arrival at a planet and departure from planet.

Therefore the procedure provides the launch date that minimise time discontinuities between phases. The lower are the discontinuities in the FGS, the lower should be the propellant consumption in the final optimised solution. This procedure has been implemented into a software tool, called BS1 in the following. The software computes one set $S^{(i)}$ of initial times $t^{(i)}_{Initial}$ per each phase i; all the dates of a set make that phase feasible, in terms of planet encounter at the starting and ending point of the arc. It then evaluates and finds the minimum of the merit function:

$$F(t_{Launch}^{(1)},\ldots,t_{Launch}^{(N)}) = \sum_{k=1}^{N}(t_{Launch}^{(k)} - t_{Launch}^{(1)})^2 \qquad \textbf{[16.]}$$

$$t_{Launch}^{(i)} = t_{Initial}^{(i)} - \sum_{k=1}^{i-1}\Delta T^{(k)} \qquad \textbf{[17.]}$$

$$t_{Initial}^{(i)} \in S^{(i)} \qquad \textbf{[18.]}$$

where $\Delta T^{(i)}$ is the transfer time of phase i, N the total phase number. Once minima are found, qualitative comparison with other strategies can be done to estimate the less unfeasible one in terms of time discontinuity.

3.1.2 - *Multiple Sinchronous and Resonant Orbit Transfer.*

In the Jovian system, low distances and fast revolutions periods of moons raise the number of interesting strategies and the number of possible phases for each strategy. Furthermore it suggests the opportunity to exploit several synchronous encounters with jovian moons performing several gravity manoeuvres to reduce energy. The idea is to look for synchronous-resonant orbits that allow to reach the desired level of energy minimising the requirements in terms of related Δv. This could imply long mission times but reduces the need of thrust which is one of mission requirements since electric propulsion can provide only small corrections and not major orbit changes in reasonable time at Jupiter distance from the Sun.

The developed procedure, implemented into a software tool, called BS2 in the following, minimise the total transfer time of synchronous orbit tours. Initial conditions for each tour of jovian moons have been considered fixed and known from final conditions of interplanetary transfer. Now for a planet-synchronous orbit the following relations hold:

$$nT = mT_P \qquad \textbf{[19.]}$$

$$\Rightarrow a = \left(\frac{n}{m}\right)^{\frac{2}{3}} a_P \qquad \textbf{[20.]}$$

where n and m are respectively the numbers of planet and spacecraft revolutions, a is the semi-major axis of the synchronous orbit, a_P is the semimajor axis of the planet and the absolute velocity at planet encounter $\boldsymbol{v}_{Abs}$ is defined as:

$$\left\|\mathrm{v}_{Abs}\right\| = f\left(\frac{n}{m}\right) = \sqrt{\frac{2\mu}{a_P}}\sqrt{1-\frac{1}{2}\left(\frac{n}{m}\right)^{-2/3}} \qquad \textbf{[21.]}$$

And for a planet-synchronous tour, assuming that the (k) fly-by (GA(k)) links the (k-1) and (k) orbits:

$$n_{TOT}^{(k)} = \sum_{i=1}^{k} n^{(i)} \qquad \textbf{[22.]}$$

$$\eta_{GA}^{(k)}\left(n^{(k)},m^{(k)}\right) = \frac{\beta_{GA}^{(k)}\left(n^{(k)},m^{(k)}\right)}{\beta_{\max GA}} \qquad \textbf{[23.]}$$

$$\beta_{GA}^{(k)} < \beta_{maxGA} \qquad \textbf{[24.]}$$

$$\beta_{GA\ TOT}^{(k)} = \sum_{i=1}^{k}\beta_{GA}^{(k)} \qquad \textbf{[25.]}$$

$$\alpha^{(k)}_{TOT} = \alpha_0 + \beta^{(k)}_{GA\ TOT} \qquad \textbf{[26.]}$$

$$\boldsymbol{v}^{(k)}_{\boldsymbol{Abs\ Syn}} = \left[-\tilde{v}\sin\alpha^{(k)}_{TOT} \quad v_P + \tilde{v}\cos\alpha^{(k)}_{TOT}\right] \qquad \textbf{[27.]}$$

where β_{maxGA} is the maximum β angle that is achieved with the minimum GA pericenter and , $\tilde{v}$ $\beta_{GA}^{(k)}$ is the angle obtained with GA(k), $\alpha^{(k)}_{TOT}$ is the angle between $\tilde{v}$ and v_P after GA(k), $\eta^{(k)}_{GA}$ is the ratio ($\beta_{GA}^{(k)} / \beta_{\max GA}$) and will be referred to as fly-by efficiency (see Figure 2).

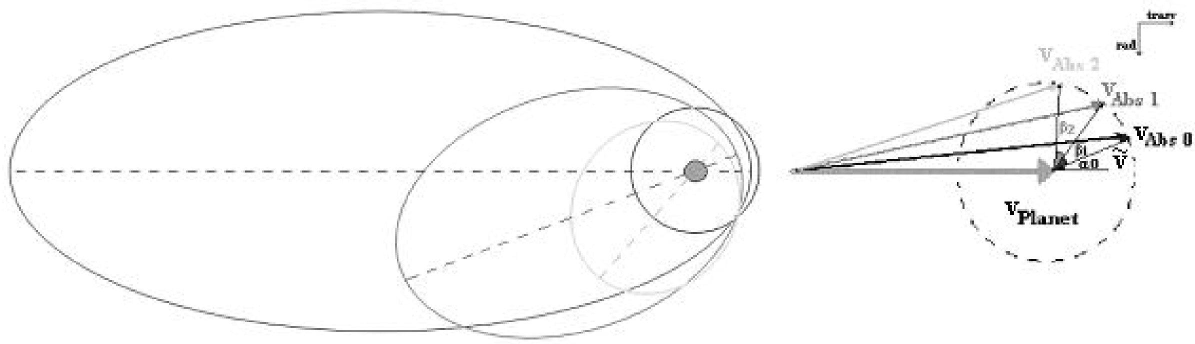


Figure 2 - First phases in a planet-synchronous tour

The problem is therefore to minimise:

$$F_{BS2}\left(N, n^{(1)}, n^{(2)}, \ldots, n^{(N)}, m^{(1)}, m^{(2)}, \ldots, m^{(N)}\right) = \sum_{k=1}^{N} n^{(k)} = n^{(N)}_{TOT} \qquad \textbf{[28.]}$$

Subject to:

$$a^{(N)} < a^{Target} \qquad \textbf{[29.]}$$

$$a^{(k+1)} < a^{(k)} \qquad \textbf{[30.]}$$

$$\beta_{GA}^{(k+1)} < \beta_{maxGA} \qquad \textbf{[31.]}$$

$$n^{(k)}_{TOT} < n_{max} \qquad \textbf{[32.]}$$

With initial and boundary conditions:

$$\mathbf{v}_{\mathbf{Abs\ 0}} \qquad \textbf{[33.]}$$

$$\mathbf{r}^{\mathbf{(k+1)}}_{\mathbf{in}} = \mathbf{r}^{\mathbf{(k)}}_{\mathbf{fin}} \qquad \textbf{[34.]}$$

$$t^{(k+1)}_{in} = t^{(k)}_{fin} \qquad \textbf{[35.]}$$

A systematic research has been then implemented using a recursive function. A high value for n_{max} is firstly guessed. During the computation, it is updated with the value of the merit function of the last optimum found. As the initial conditions are known, $\tilde{v}$ α_0 and β_{maxGA} are calculated. Then in each phase *k+1,* new inputs are $n^{(k)}/m^{(k)}$, $a^{(k)}$, and constraints [30.] [31.] can be written as:

$$\frac{n^{(k+1)}}{m^{(k+1)}} < \frac{n^{(k)}}{m^{(k)}} \qquad \textbf{[36.]}$$

$$\alpha^{(k)}_{TOT} + \beta^{(k+1)}_{GA\ \ TOT} < \alpha^{(k)}_{TOT} + \beta_{\max GA} \qquad \textbf{[37.]}$$

Since $\dfrac{\partial\left\|\boldsymbol{v}_{\boldsymbol{Abs\,Syn}}\right\|}{\partial\alpha_{TOT}} < 0$ the latter equation becomes:

$$\left\|\boldsymbol{v}_{\boldsymbol{Abs\ Syn}}\left(\alpha^{(k)}_{TOT} + \beta^{(k+1)}_{GA\ \ TOT}\right)\right\| > \left\|\boldsymbol{v}_{\boldsymbol{Abs\ Syn}}\left(\alpha^{(k)}_{TOT} + \beta_{\max GA}\right)\right\| \qquad \textbf{[38.]}$$

and using [26.] [27.]:

$$\frac{2\mu}{a_P}\left(1-\frac{1}{2}\left(\frac{n^{(k+1)}}{m^{(k+1)}}\right)^{-2/3}\right)>\tilde{v}^2+v_P^2+2v_P\tilde{v}\cos\left(\alpha_{TOT}^{(k)}+\beta_{\max GA}\right) \quad \textbf{[39.]}$$

Now manipulating equation [39.], after some algebra n/m can be obtained esplicitely as a function of α in the form:

$$\frac{n^{(k+1)}}{m^{(k+1)}}>g\left(\alpha_{TOT}^{(k)}\right) \quad \textbf{[40.]}$$

Finally, [30.] [31.] become

$$g\left(\alpha_{TOT}^{(k)}\right)<\frac{n^{(k+1)}}{m^{(k+1)}}<\frac{n^{(k)}}{m^{(k)}} \quad \textbf{[41.]}$$

This constraint can be solved only for $m^{(k+1)} > m_{MIN}$, where m_{MIN} is the lowest $m^{(k+1)}$ that guarantees the existence of an integer $n^{(k+1)}$ between ($m^{(k+1)}*g$) and ($m^{(k+1)}* n^{(k)}/m^{(k)}$). Then domains of $n^{(k+1)}$ and $m^{(k+1)}$ are

$$m_j^{(k+1)}=m_{MIN}^{(k+1)}+j \qquad j=1,L \quad \textbf{[42.]}$$

$$n_{ij}^{(k+1)}=n_{MIN\ j}^{(k+1)}+i \qquad i=1,p \quad \textbf{[43.]}$$

where

$$n_{MIN\ j}^{(k+1)}=\underset{\to+\infty}{round}\left(g\left(\alpha_{TOT}^{(k)}\right)m_j^{(k+1)}\right) \quad \textbf{[44.]}$$

$$p=\min(\underset{\to 0}{round}\left(\frac{n^{(k)}}{m^{(k)}}m_j^{(k+1)}\right),M)-n_{MIN\ j}^{(k+1)} \quad \textbf{[45.]}$$

and L, M are the highest integers that satisfy [32.], while rounding towards +∞ or towards 0 is necessary to keep the integer ratio between the constraints.

The number of feasible solutions can be roughly estimated as $(Lp)^N$, therefore a systematic search would imply calculating some $(100)^{10}$ possible paths. Since the number of variables grows up exponentially with N, a way to reduce the number of possible solutions has been investigated, by looking for the most efficiency sequence. This can be achieved looking for highly efficient flyby, i.e. optimum η_{GA}, or choosing, in each phase, the orbit that minimizes the transfer time to the next planet encounter. In the latter case, the number of solutions is reduced down to only one and each phase is simply determined by:

$$m^{(k+1)}=m_{MIN}^{(k+1)}; \qquad n^{(k+1)}=n_{MIN\ j=0}^{(k+1)} \quad \textbf{[46.]}$$

Therefore each phase is a local optimum; however, this does not guarantee that the correspondent solution is the global optimum. Then the efficiency of each flyby has been maximised by evaluating the gradient of η. From [26.] [27.], and using [21.]:

$$\eta_{GA}^{(k+1)}\left(n^{(k+1)},m^{(k+1)}\right)=\left[\operatorname{acos}\left(\frac{v_{Abs\quad Syn}^{(k+1)}\left(n^{(k+1)},m^{(k+1)}\right)-\tilde{v}^2-v_P^2}{2v_P\tilde{v}}\right)-\alpha_0-\sum_{i=1}^{k}\beta_{GA}^{(i)}\right]/\beta_{\max GA} \quad \textbf{[47.]}$$

Now after some algebra the gradient of the efficiency parameter η can be written as:

$$\nabla\eta_{GA}^{(k+1)}\left(n^{(k+1)},m^{(k+1)}\right)=-K\nabla\left(n^{(k+1)}/m^{(k+1)}\right)=K\left[-\frac{1}{m^{(k+1)}}\quad+\frac{n^{(k+1)}}{\left(m^{(k+1)}\right)^2}\right] \quad \textbf{[48.]}$$

where K is a positive quantity. As a result we obtain the following conditions:

$$\begin{cases} \dfrac{\partial \eta_{GA}^{(k+1)}}{\partial n^{(k+1)}} < 0 \\ \dfrac{\partial \eta_{GA}^{(k+1)}}{\partial m^{(k+1)}} > 0 \end{cases} \qquad \textbf{[49.]}$$

High η can then be achieved by increasing m or decreasing n. Since increasing m also implies augmenting transfer time, then only the minimum value for n is taken, introducing the following additional constraint:

$$n_j^{(k+1)} = n_{MIN\ j}^{(k+1)} \qquad \textbf{[50.]}$$

In this way the number of variables is reduced from one infinite set to one infinite countable set:

$$r_{GApericeter} \xrightarrow[\textit{model}]{\textit{Synchronous orbit}} n, m \xrightarrow[\textit{model}]{\textit{high GA efficiency}} m$$

Still, the CPU time could be too high. However, using a recursive algorithm the number of function calls has been drastically reduced, and the CPU time is of the order of few seconds. A special case has been considered, when the target of a synchronous tour is to reach a planet. In this case the relative velocity to the target planet has to be minimised. To achieve this result the outgoing absolute velocity of the spacecraft after the last gravity manoeuvre must be such that to minimise the access velocity to the target planet, i.e. to minimise:

$$\|\tilde{\boldsymbol{v}}_{\boldsymbol{B}}\| = f(\boldsymbol{v}_A) \qquad \textbf{[51.]}$$

subject to

$$\|\tilde{\boldsymbol{v}}_{\boldsymbol{A}}\| = \textit{fixed} \qquad \textbf{[52.]}$$

$$r_{\text{transfer orbit pericenter}} \leq a_B \qquad \textbf{[53.]}$$

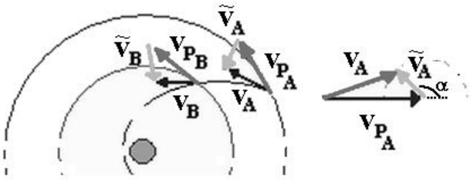


Figure 3 - Special case: reaching a new planet

The problem can be solved in the following way, since:

$$\mathbf{v_A} = \mathbf{g}(\alpha) \qquad \textbf{[54.]}$$

$$\alpha_{min} < \alpha < \pi \qquad \textbf{[55.]}$$

$$v_A^2 = (v_{PA}^2 + \tilde{v}_A^2) - 2\, v_{PA}\, \tilde{v}_A \cos\alpha \qquad \textbf{[56.]}$$

$$v_{RelB}^2 = (v_{PB} - v_{Btrasv})^2 + (v_B^2 - v_{Btrasv}^2) \qquad \textbf{[57.]}$$

where α_{min} corresponds to the an orbit tangent to the orbit of the target planet, then:

$$\frac{\partial(\tilde{v}_B^2)}{\partial\alpha} = \frac{\partial}{\partial\alpha}\left(v_B^2 - 2v_{PB}v_{Btrasv}\right) = \frac{\partial}{\partial\alpha}(2E) - 2v_{PB}\frac{\partial}{\partial\alpha}\left(\frac{h}{r_{PB}}\right) = \frac{\partial}{\partial\alpha}\left(v_A^2\right) - 2v_{PB}\frac{r_{PA}}{r_{PB}}\frac{\partial}{\partial\alpha}\left(v_{Atrasv}\right) =$$

$$2v_{PA}\tilde{v}_A\frac{\partial}{\partial\alpha}(\cos\alpha) - 2v_{PB}\frac{r_{PA}}{r_{PB}}\tilde{v}_A\frac{\partial}{\partial\alpha}(\cos\alpha) = 2\tilde{v}_A\sin\alpha\left(v_{PB}\frac{r_{PA}}{r_{PB}} - v_{PA}\right) = 2\tilde{v}_A r_{PA}\sin\alpha\left(\frac{\sqrt{\mu/r_{PB}}}{r_{PB}} - \frac{\sqrt{\mu/r_{PA}}}{r_{PA}}\right) = K\sin\alpha$$

$$\text{for } K > 0 \quad \Rightarrow \quad \frac{\partial(\tilde{v}_B^2)}{\partial\alpha} > 0 \quad \Rightarrow \quad \min_{\alpha}\tilde{v}_B^2 = \tilde{v}_B^2(\alpha_{\min})$$

[58.]

The minimum $\tilde{v}_B$ is obtained when the inequality constraint is active, i.e. when the final transfer orbit is tangent to the orbit of the target planet.

3.1.3 - *Optimal Impulsive Transfer Trajectories.*

From the experience gained with BS1 a third procedure has been developed and implemented in the software tool called in the following BS3. The new procedure implements a more sophisticated model for planet encounters including a link-conic model for swing-bys. This third procedure has been used for chemical propulsion option and as a FGS for electric propulsion options. In fact the idea is that impulsive solutions can be regarded as the minimum necessary Δv required for a given transfer. The sequence that minimise this Δv will also minimise the need for low thrust and therefore propellant consumption of the electric propulsion option.

In the BS3 physical model, planet orbits are no more coplanar nor circular (even though perturbation effects are still neglected). Each coast arc between two subsequent encounters are split in two parts. The three components of the position vector at the beginning of one part are then forced to be equal the three components of the position vector of the previous part. No constraint is forced on velocity components allowing, in this way, an impulsive **Δv** manoeuvre between the two parts. Each arc has been designed solving a Lambert problem with Battin's approach[4].

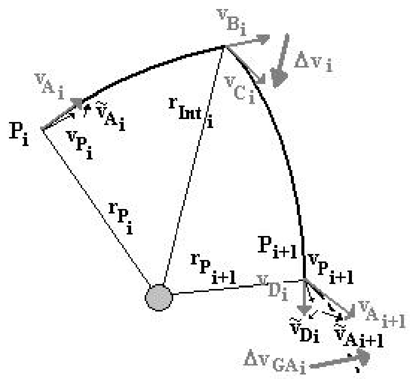


Figure 4 - BS3 physical model.

The problem is therefore to find the date of each planet encounter , the launch and arrival dates, and the points in space and time where to perform a manoeuvre in order to minimise the sum of all required **Δv**. The trajectory, that minimise fuel consumption from planet P_1 to P_N makes use of N-1 impulsive manoeuvres and N-2 flybys therefore the state vector on the resulting NLP problem has (6N-6) components:

$$\mathbf{X}= [t_{P(1)},...,t_{P(N)}, t_{Int(1)}, \dots, t_{Int(N-1)}, \mathbf{r}_{\mathbf{Int(1)}}, \dots, \mathbf{r}_{\mathbf{Int(N-1)}}, r_{\pi(1)}, \dots, r_{\pi(N-2)}]^T \quad \textbf{[59.]}$$

Where $t_{P(i)}$ is the time of planet (i) encounter, $t_{Int(i)}$ is the time of impulsive manoeuvre (i), $\mathbf{r}_{\mathbf{Int(i)}}$ is the position of impulsive manoeuvre (i), $r_{\pi(1)}$ is the pericenter of GA manoeuvre (i).

With a given **X**, planet positions (and then the orbiter positions) at encounter times are evaluated, while velocities are calculated solving the Lambert problem.

The problem is then to minimise the objective function:

$$F_{BS3}(\mathbf{X})=\sum_{i=1}^{N-1}\left\|\Delta\mathbf{v}^{(i)}\right\|=\sum_{k=1}^{N-1}\left\|\mathbf{v_C}^{(i)}-\mathbf{v_B}^{(i)}\right\| \quad \textbf{[60.]}$$

subject to

$$||\tilde{v}_{\mathbf{D(i)}}||=||\tilde{v}_{\mathbf{A(i+1)}}|| \quad \textbf{[61.]}$$

$$<\tilde{v}_{\mathbf{D(i)}},\tilde{v}_{\mathbf{A(i+1)}}>= ||\tilde{v}_{\mathbf{D(i)}}||||\tilde{v}_{\mathbf{A(i+1)}}||*\cos(2\beta) \quad \textbf{[62.]}$$

Other constraints have been included as:

$$\|\tilde{\boldsymbol{v}}_{\mathbf{A(1)}}\| < \tilde{v}_{\mathrm{MaxIN}}\,,\ \|\tilde{\boldsymbol{v}}_{\mathbf{A(N)}}\| < \tilde{v}_{\mathrm{MaxFIN}} \qquad \textbf{[63.]}$$

in order to limit departure escape velocity and arrival relative velocity at Jupiter.

3.2 - DIRECT OPTIMISATION

A general trajectory design problem can be decomposed in M phases, each one characterised by a time domain D^J ,with $j=1,..,M$, a set of m dynamic variables $\mathbf{x}$, a set of n control variables $\mathbf{u}$ and a set of l parameters $\mathbf{p}$. Furthermore, each phase j may have an objective function

$$J^j = \phi^j(\mathbf{x}_0^b, \mathbf{x}_f^b, t_f, \mathbf{p}) + \int_{t_i}^{t_f} L^j(\mathbf{x},\mathbf{u},\mathbf{p})dt \qquad \textbf{[64.]}$$

a set of dynamic equations

$$\dot{\mathbf{x}} - \mathbf{F}^j(\mathbf{x},\mathbf{u},\mathbf{p},t) = 0 \qquad \textbf{[65.]}$$

a set of algebraic constraints on states and controls

$$\mathbf{G}^j(\mathbf{x},\mathbf{u},\mathbf{p},t) \geq \mathbf{0} \qquad \textbf{[66.]}$$

and a set of boundary constraints

$$\psi^j(\mathbf{x}_0^b, \mathbf{x}_f^b, \mathbf{p}, t)\Big|_{t_0}^{t_f} \geq 0 \qquad \textbf{[67.]}$$

Among boundary constraints a set of inter-phase link constraints exist that are used to assemble all phases together

$$\psi^j(\mathbf{x}_j^b, \mathbf{x}_{j-1}^b, \mathbf{p}, t) \geq 0 \qquad \textbf{[68.]}$$

The time domain $D(t_0,t_f)\subset\Re$ relative to each phase j can be further decomposed into N finite time elements $D^j = \bigcup_{i=1}^N D_i^j(t_{i-1},t_i)$ and, on each time element D^j_i, states and controls $[\mathbf{x},\mathbf{u}]$ can be parameterised as follows:

$$\begin{Bmatrix}\mathbf{x}\\ \mathbf{u}\end{Bmatrix} = \sum_{s=1}^{p} f_s(t)\begin{Bmatrix}\mathbf{x}_s\\ \mathbf{u}_s\end{Bmatrix} \qquad \textbf{[69.]}$$

where the basis functions f_s are chosen within the space of polynomials of order $p-1$:

$$f_s \in P^{p-1}(D_i^j) \qquad \textbf{[70.]}$$

Therefore in general a finite element is defined by a sub-domain D^j_i, and by a sub-set of parameters $[\mathbf{x}_s,\mathbf{u}_s,\mathbf{p}]$. A group of finite elements forms a phase and a group of phases forms the original optimisation problem. Notice that additional parameters $\mathbf{p}$ may occur in all constraint equations depending on their function in the optimisation problem. Furthermore it should be noticed that each phase can be grouped in sequence or in parallel with the other phases depending on its time domain and on the inter-phase link constraints that pass information among phases. Thus two phases can share the same time domain but have different parameterisations.

Now taking a general phase, in order to integrate differential constraints [65.], on each finite element i, differential equations are transcribed into a weighted residual form considering boundary conditions of the weak type:

$$\int_{t_i}^{t_{i+1}} \left\{\dot{\mathbf{w}}^T\mathbf{x} + \mathbf{w}^T\mathbf{F}^j\right\}dt - \mathbf{w}_{i+1}^T\mathbf{x}_{i+1}^b + \mathbf{w}_i^T\mathbf{x}_i^b = 0 \qquad i = 1,\ldots,N-1 \qquad \textbf{[71.]}$$

where **w**(t) are generalised weight (or test) functions defined as:

$$\mathbf{w} = \sum_{s=1}^{p+1} g_s(t)\mathbf{w}_s \qquad [72.]$$

where g_s are taken within the space of polynomials of order p:

$$g_S \in P^p(D_i^j) \qquad [73.]$$

Now the problem is to find the vector $\mathbf{x}_s \in \Re^{p*m}$,the vector $\mathbf{u}_s \in \Re^{p*n}$, the vector $\mathbf{p} \in \Re^l$ and $\mathbf{x}^b_f$ and $\mathbf{x}^b_0 \in \Re^m$ that satisfy variational equation [71.] along with algebraic and boundary constraints:

$$\mathbf{G}^j(\mathbf{x}, \mathbf{u}, \mathbf{p}, t) \geq 0 \qquad [74.]$$

$$\psi^j(\mathbf{x}_0^b, \mathbf{x}_f^b, \mathbf{p}, t)\Big|_{t_0}^{t_f} = 0 \qquad [75.]$$

where quantities $\mathbf{x}_s$, and $\mathbf{u}_s$ are called internal node values, while $\mathbf{x}^b_f$, $\mathbf{x}^b_0$ are called boundary values. Notice that generally the order p of the polynomials can be different for states and controls.

Each integral of the continuous forms [64.]and [71.] is then replaced by a q-points Gauss quadrature sum, where q is taken equal to p. For continuous solution, in order to preserve the continuity of the states, at matching points, the following condition must hold:

$$\mathbf{x}^b_i = \mathbf{x}^b_{i+1} \qquad i=1,\ldots,N-2 \qquad [76.]$$

Thus all the boundary quantities [76.] cancel one another except for those at the initial and final times. Algebraic constraint equation [74.] can be collocated directly at Gauss nodal points:

$$\mathbf{G}^j{}_s(\mathbf{x}_s(\xi_s), \mathbf{u}_s(\xi_s), \mathbf{p}, \xi_s) \geq 0 \qquad [77.]$$

The resulting set of non-linear algebraic equations, assembling all the phases, along with discretised objective function [74.] can be seen as a general non-linear programming problem (NLP) of the form:

$$\min \ J(\mathbf{y}) \qquad [78.]$$

subject to

$$\begin{aligned} &\mathbf{c}(\mathbf{y}) \geq 0 \\ &\mathbf{b}_l \leq \mathbf{y} \leq \mathbf{b}_u \end{aligned} \qquad [79.]$$

where, $\mathbf{y}=[\mathbf{x}_s,\mathbf{u}_s,\mathbf{x}^b_0,\mathbf{x}^b_f,t_0,t_f,\mathbf{p}]$is the vector of NLP variables, J(**y**) the objective function to be minimised, **c**(**y**) a vector of non-linear constraints and $\mathbf{b}_l$ and $\mathbf{b}_u$ respectively lower and upper bounds on NLP variables.

4 - MISSION DESIGN

Some trade-offs were done in order to fix the type and number of thrusters and the type and dimension of solar arrays. The thrust level needed while approaching Jupiter was the dimensioning requirement. Then power needed at EoL at Jupiter has been estimated using the specific thrust of ion engines. Afterwards, power needed at BoL at 1 A.U. was used to calulate solar array dimension and mass. A trade-off between High-efficiency Si and Triple junction GaAs brought us to choose for the latter, combined with concentrator, in order to reduce panel size. The final configuration came out with seven 150mN ion thrusters.

Launcher performances has been also considered: an estimate of 1500 kg at launch suggests that trade-off could be done between Soyuz/Fregat and Dnepr/Varyag. The former was chosen, since it achieve higher mass at launch with the same C3 (3.16 km^2/s^2). The final data are summarised in the next table, for two different thrust level.

4.1 - EARTH TO JUPITER TRANSFER

Venus was chosen as first GA planet, since reaching it requires lower Δv than reaching other planets. Afterwards a strategy to raise apocenter and pericenter was build up, within the hypothesis of BS1. Thereby the GA sequence came out simply as EVMEJ; however BS1 found out that the optimum solution for this sequence still has great discontinuities, and suggested the introduction of a phasing orbit before the encounter with Venus. With this strategy BS1 showed that only two optimal first guess solutions in twenty years are available.

Since Mars swing-by appears to be scarcely efficient and reduces the number of launch opportunities a second option has been optimised from the preceding one, without Mars encounter. Furthermore, in order to investigate the possibility to minimise solar array size, two different power requirements have been considered for the two options: 50 kW at 1 AU for the first one and 40 kW at 1 AU for the second one.

4.2 - CAPTURE

A brief study has been developed to find out which planet should be used to gain the necessary Δv to be captured. Ganymede was chosen, since it has the greater dominium of possible initial conditions that guarantee the capture as shown in figure 5 where apocenter and period of possible orbits after the first GA are shown, as a function of $v_{sphere\ of\ infl}$ and α_0 , the angle between $\mathbf{v_{Abs}}$ and $\mathbf{v_{Planet}}$. GA pericenter is 300 km above the planet surface. In this trade-off, as in following ones, the possibility to choose Io as a flyby planet was rejected in order to reduce the orbiter exposure to Jupiter radiation.

Furthermore looking at the period of the first orbit after capture Ganymede appears also the best planet for the first synchronous orbits as shown in figure 6.

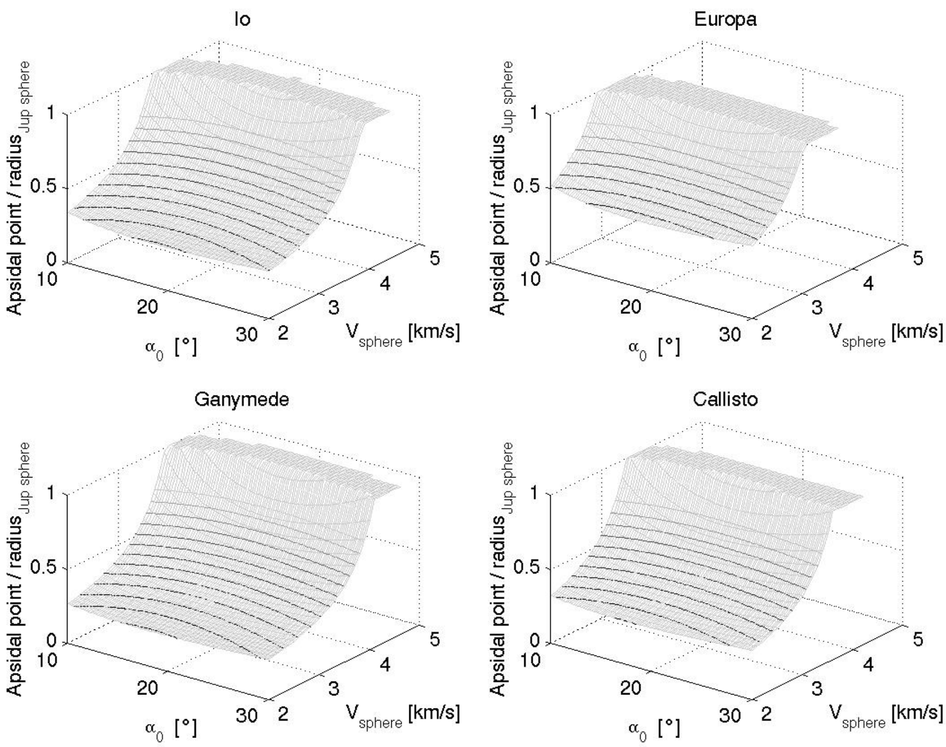


Figure 5 - Possible apocenters of the first orbit

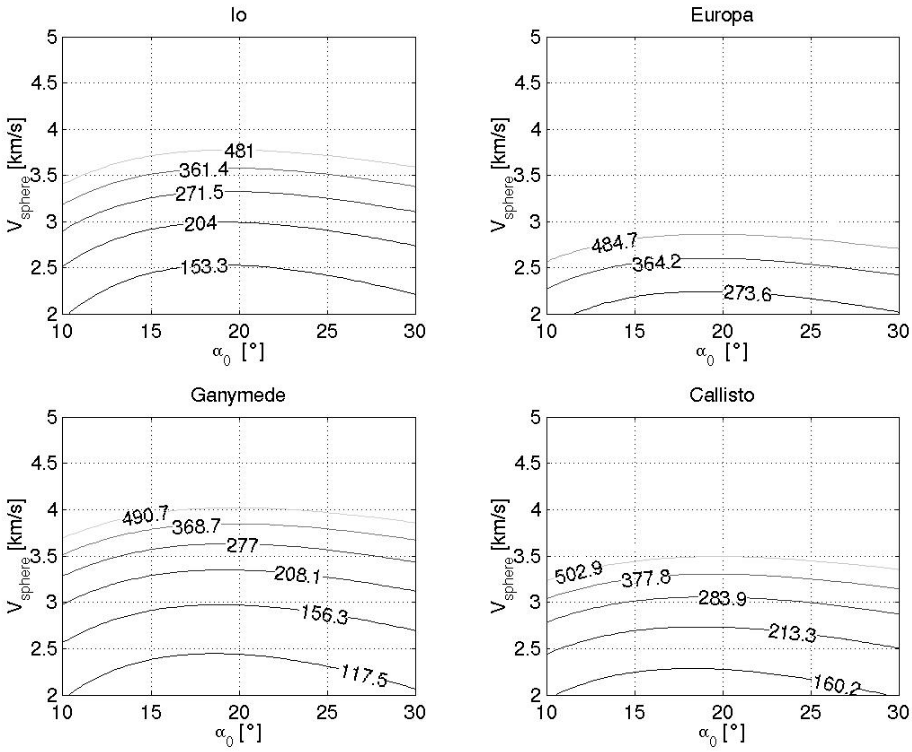


Figure 6 - Possible period of the first orbit

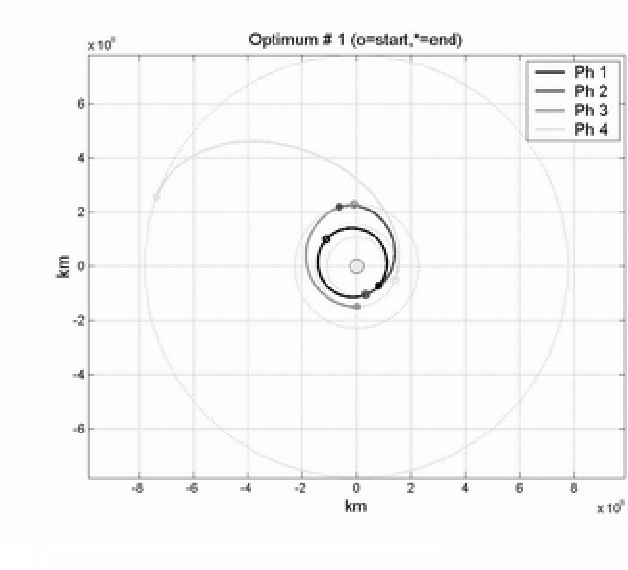
4.3 - Resonant Descent to Europa

In the planar circular model, the lowest velocity relative to Europa ($\tilde{v}_{Eu}$ = 1.49 km/s) can be achieved by a Hohmann transfer from Ganymede ($H_{Ga\text{-}Eu}$). However the corresponding relative velocity at Ganymede ($\tilde{v}_{Ga}$=1.33 km/s) cannot be achieved using Io, since the minimum $\tilde{v}_{Ga}$, corresponding to a Hohmann transfer from Io to Ganymede ($H_{Io\text{-}Ga}$), is too high (2.70 km/s) and the radiation level related to an Io sequence is too high. Another option that makes use of a $H_{Ca\text{-}Ga}$ has been investigated as a possible solution, since it gives a $\tilde{v}_{Ga}$ that is slightly above the requested ($\tilde{v}_{Ga}$ = 1.41 km/s). However, in order to have the corresponding $\tilde{v}_{Ca}$ (1.22 km/s), the only solution is to use Ganymede to lower the initial energy and then make a 0.8 km/s Δv manoeuvre. Thus other solutions have been investigated. Since transfers that are tangent to target planet guarantee minimum relative velocity, a strategy involving synchronous tour leading to such tangent-to-target transfer has been implemented. The planet sequence is made up by a synchronous Ganymede tour till a tangent orbit to Europa, then an Europa tour to reach a tangent orbit to Ganymede, and finally a Ganymede-Europe tangent orbit.

5 - RESULTS

5.1 - Transfer Solutions

At first the effectiveness of the proposed approach to generate an optimal first guess solution was assessed analysing the transfer trajectory to Jupiter only. For this test cases the EVMEJ sequence have been considered with the additional phasing orbit, a mass at launch M_{Launch}=2280 kg and a C3=6.25 km/s. Only the best two solutions generated by BS1 were optimised and in particular the most interesting one was optimised both considering electric propulsion and chemical propulsion (with an Isp=315 s). The results obtained are represented in figures 7 to 9.



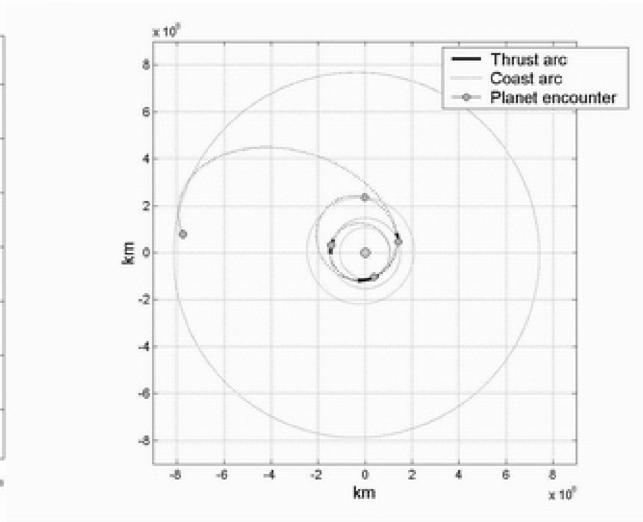


Figure 7 - Solution number 1 : FGS generated by BS1 (left) and optimised with electric propulsion (right).

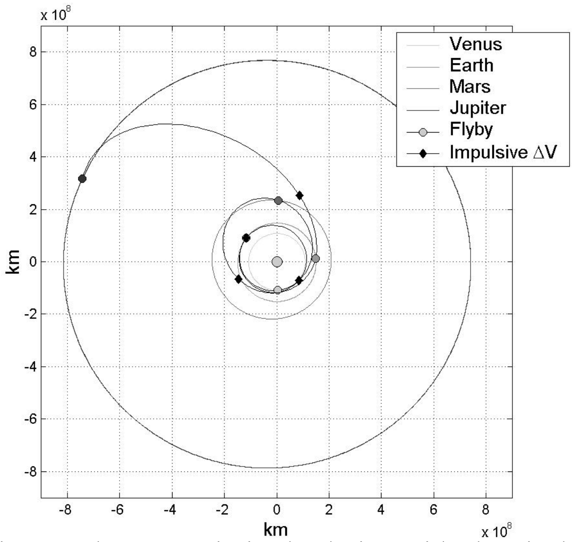
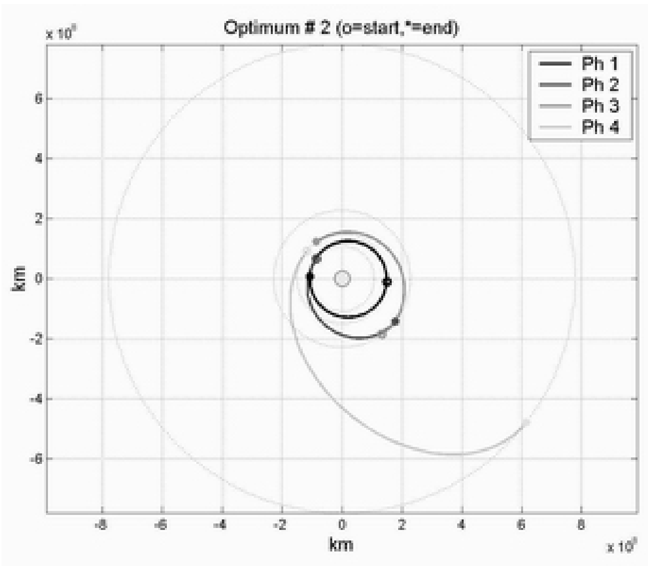


Figure 8 - Solution number 1 : optimised solution with chemical propulsion



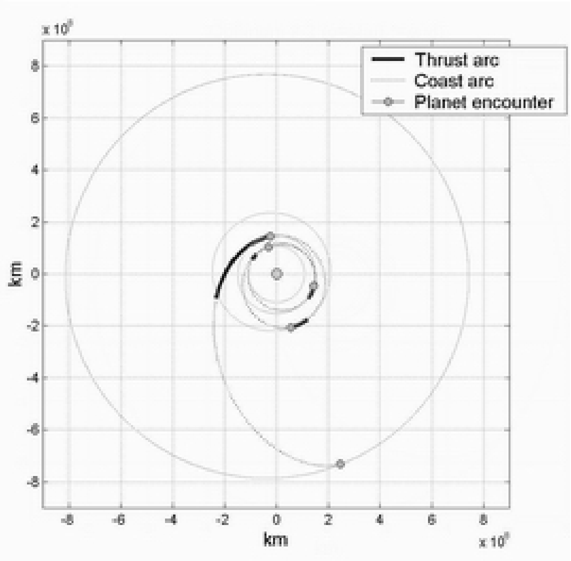


Figure 9 - Solution number 2 : FGS generated by BS1 (left) and optimised with electric propulsion (right).

Table 2. Optimal solution 1

GA PLANET	FIRST GUESS [MJD]	OPTIMISED [MJD]	CHEMICAL[MJD]
Earth	Dep (Phase 1): 3692	Dep:3719	Dep: 3690
Venus	Arr (Phase 1): 4130 Dep (Phase 2): 4110	Flyby: 4112	Flyby: 4101
Mars	Arr (Phase 2): 4328 Dep (Phase 3): 4299	Flyby: 4284	Flyby: 4280
Earth	Arr (Phase 3): 4558 Dep (Phase 4): 4628	Flyby: 4667	Flyby: 4656
Jupiter	Arr (Phase 4): 5625	Arr: 5962	Arr: 5770
PROPELLANT MASS		111 kg	1465 kg

Table 3. Optimal solution 2

GA PLANET	FIRST GUESS [MJD]	OPTIMISED [MJD]
Earth	Dep (Phase 1): 1356	Dep:1343
Venus	Arr (Phase 1): 1794 Dep (Phase 2): 1773	Flyby: 1751
Mars	Arr (Phase 2): 1990 Dep (Phase 3): 1959	Flyby: 1942
Earth	Arr (Phase 3): 2218 Dep (Phase 4): 2234	Flyby: 2190
Jupiter	Arr (Phase 4): 3232	Arr: 3128
PROPELLANT MASS		183 kg

5.2 - TOUR DESIGN

The tour of jovian moons has been designed using BS2 to estimate the best sequence of swing-bys. Initial conditions have been derived from the transfer trajectory. The result obtained has been represented in figure 10. The same sequence estimated using BS2 has then been optimised using DFET and the result has been reported in table 4.

Since the real motion of planet and the mean one are slightly different, an error of 0.15% is obtained in evaluating distance from Jupiter while approaching Ganymede. Such a small error is amplified while evaluating semi-major axis. The a error is 5-6% for a high energy orbit (similar to the first one), and decrease to 0.5-0.6% for low energy orbit (like the last one). This non-linear behaviour justifies errors obtained in the prediction of the first orbits.

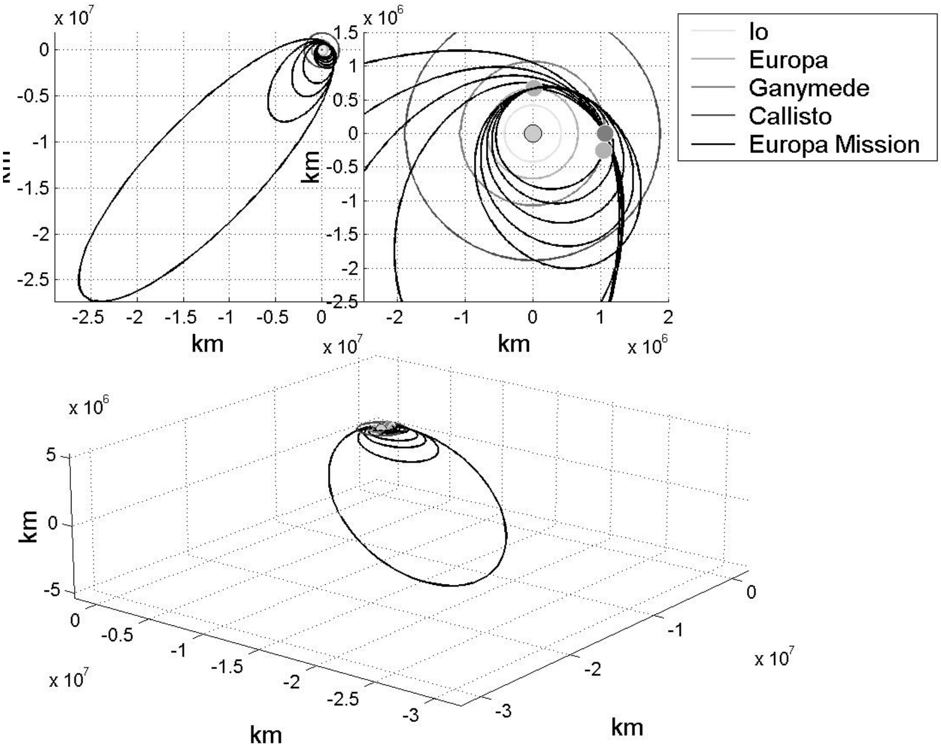


Figure 10 - FGS as found by BS2.

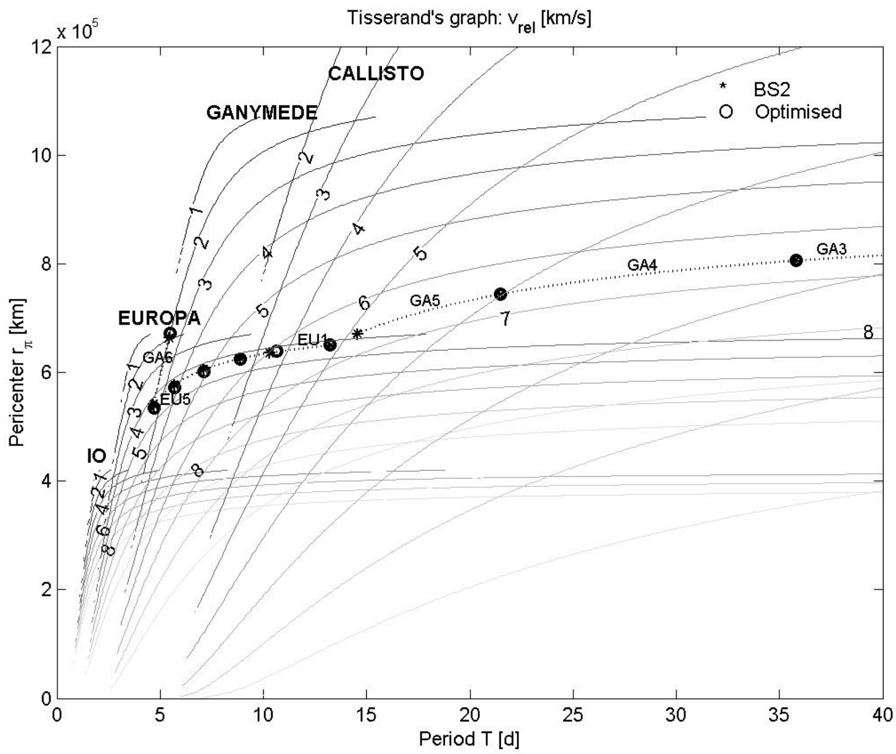


Figure 11 - Tour representation on Tisserand's plane

As a further confirmation of the effectivness of the strategy implemented in BS2, the designed tour of jovian satellites has been represented on Tisserand's plane[3] (figure 11), along with the optimised sequence, presenting a remarkable agreement and thus suggesting that BS2 could be used as an alternative to the Tisserand's plane itself. It should be noticed that the tour proposed here does not take into account any constraint on the radiation dose. Anyway using BS2, many other optimal sequences, minimising the total dose, can be easily generated. The distinct advantage of this sequence is that it allows to reach Europa with a low relative velocity of 1.65 km/s (optimised solution).

Table 4. BS2 validation

PHASE NUMBER	APOCENTER (FGS)	APOCENTER (OPTIMISED)	RESONANCE N:M (FGS)	RESONANCE N:M (OPTIMISED)
1	1.90e7 km	1.80e7 km	75:1	69:1
2	4.97e6 km	5.92e6 km	10:1	7:1
3	3.13e6 km	3.19e6 km	5:1	5:1
4	2.22e6 km	2.22e6 km	3:1	3:1

5.3 - OPTIMAL ASSEMBLED SOLUTION

Now the two parts of the trajectory, the transfer phase and the tour phase, have been assembled together to form a single NLP problem and a single trajectory as mantioned above and optimised. At the entrance of the sphere of influence of Jupiter gravity perturbations due to the Sun have been taken into account as, while approaching Jupiter, gravity perturbations due to the giant planet have been considered. The resulting optimal solution have been represented in Figure 16 -for the transfer phase and in Figure 17 -for the jovian tour representing thrust arcs with a solid line and coast arcs with a dashed line.

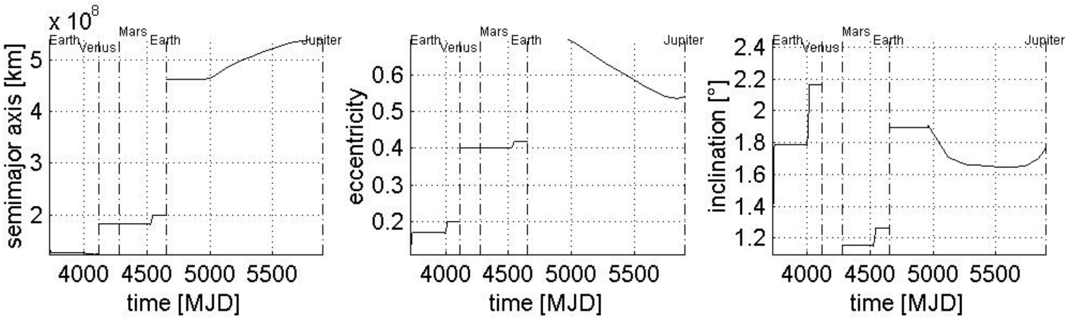


Figure 12 - Main parameters and variables : interplanetary transfer

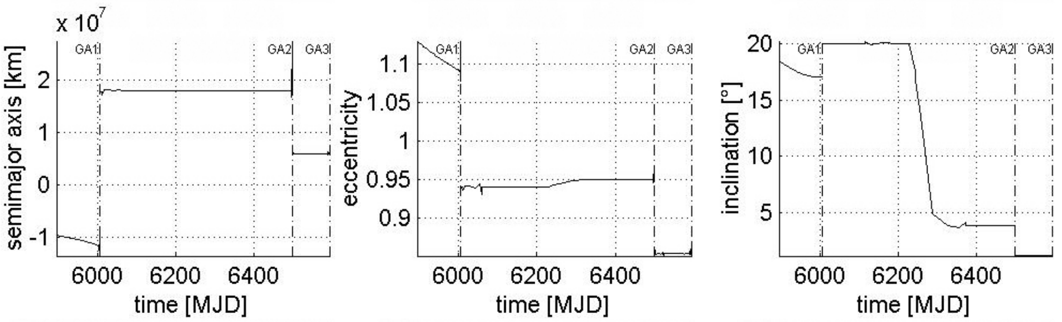


Figure 13 - Main parameters and variables capture

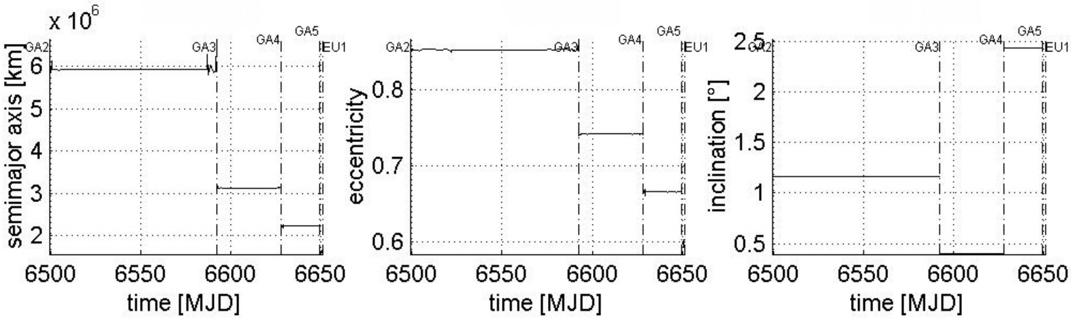


Figure 14 - Main parameters and variables : Ganymede tour

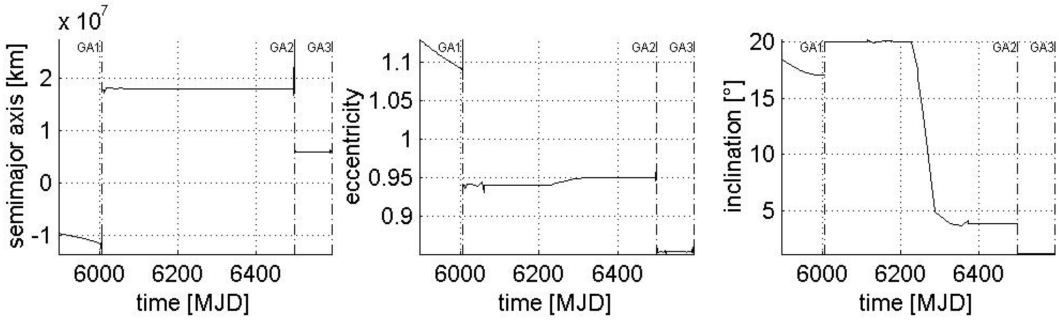


Figure 15 - Main parameters and variables : Europe tour

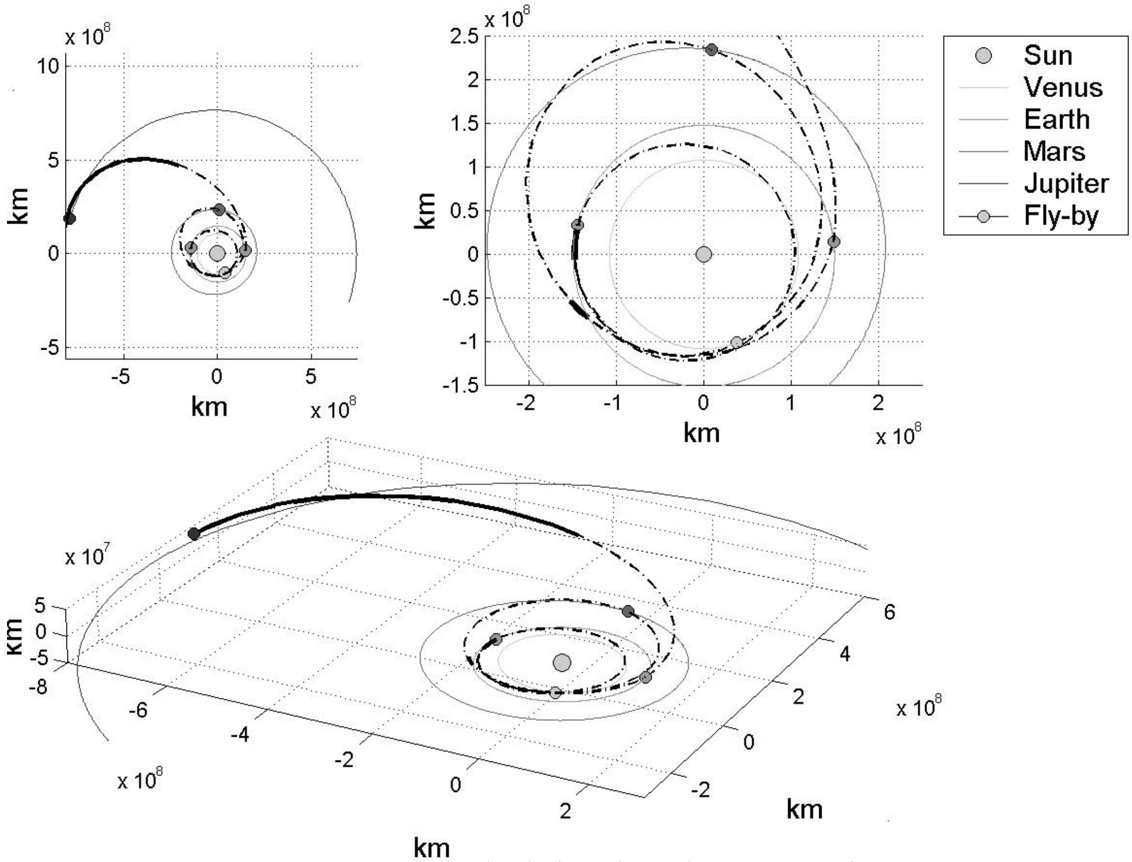


Figure 16 - Optimised solution : interplanetary transfer

In Figure 12 - to Figure 15 - the time history of semimajor-axis, inclination and eccentricity have been represented for the transfer trajectory, the capture and the complete tour, showing the effect of each swing-by. Finally in table **5** the mass budget for both options have been summarized considering the power provided by the solar arrays at 1AU at the end of life (EOL).

Table 5. Summarizing table for two alternative options

REQUIRED POWER	40kW(EOL)	50 kW(EOL)
Solar array area	133 m^2	162 m^2
Ion Thruster	49 kg	49 kg
PPU	97 kg	97 kg
Tank+Harness+Piping	30 kg	30 kg
Flux lines	15 kg	15 kg
CU	5 kg	5 kg
Propellant	130 kg	160 kg
Solar Array	416 kg	506kg
TOTAL SEP mass	773 kg	863 kg
Launcher perfo (C3=3.16 km^2/s^2)	1500 kg	1500 kg

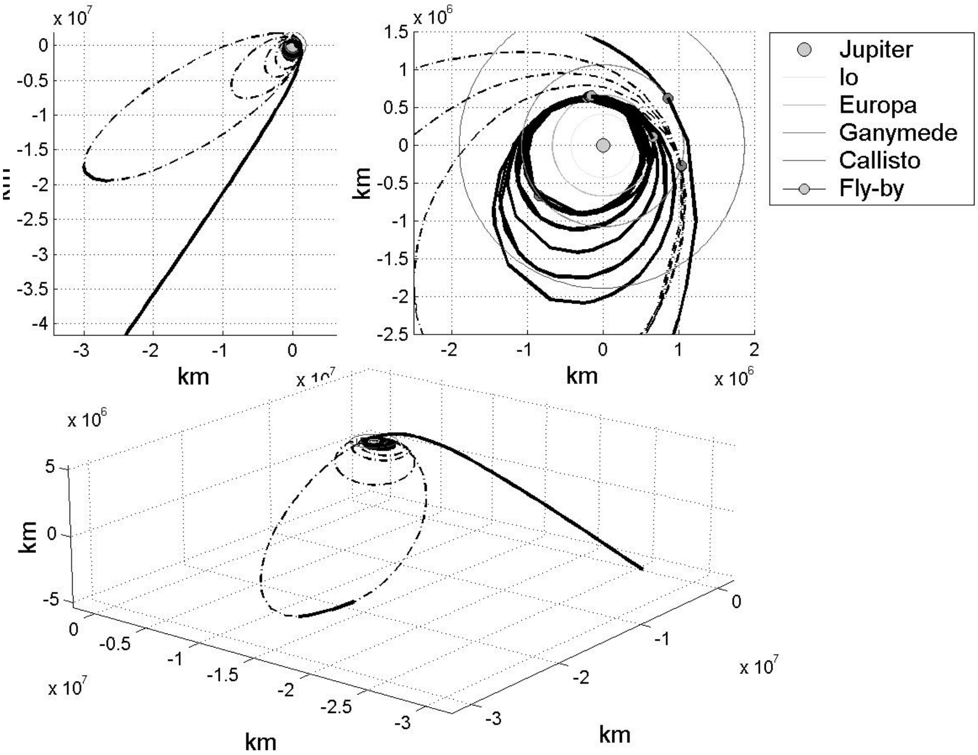


Figure 17 - Optimised solution : Jovian system

6 - CONCLUSIONS

In this paper a mission to Europa has been designed considering electric propulsion as main source of thrust. This difficult task has been accomplished using different optimisation tools: a procedure to find a global optimum for multiple gravity assist trajecotries and a general optimisation tool for optimal trajectory design based on direct transcription. The global search was extremelly effective in providing a good first guess for the following optimisation and in particular to estimate correctly the optimal tour of jovian moons. The resulting solution shows how a mission to Europa using solar electric propulsion could be feasible and interesting. However several problems are still open and deserve a further investigation, in particular the total dose of radiation for the designed tour, but many other efficient tours can be easily designed, and the final capture into a stable orbit around Europa which still requires a chemical manouvre.

7 - BIBLIOGRAPHY

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